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Quasinormal modes of plane-symmetric black holes according to the AdS/CFT correspondence

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Abstract: The electromagnetic and gravitational quasinormal spectra of (3 + 1)-dimensional plane-symmetric anti-de Sitter black holes are analyzed in the context of the AdS/CFT correspondence. According to such a correspondence, the electromagnetic and gravitational quasinormal frequencies of these black holes are associated respectively to the poles of retarded correlation functions of R-symmetry currents and stress-energy tensor in the holographically dual conformal field theory: the (2+1)-dimensional $\mathcal{N} = 8$ super-Yang-Mills theory. The connection between AdS black holes and the corresponding field theory is used to unambiguously fix the boundary conditions that enter the proper definition of quasinormal modes. Such a procedure also helps one to decide, among the various different possibilities, what are the appropriate gauge-invariant quantities one should use in order to correctly describe the electromagnetic and gravitational blackhole perturbations. These choices imply in different dispersion relations for the quasinormal modes when compared to some of the results in the literature. In particular, the long-distance, low-frequency limit of dispersion relations presents the characteristic hydrodynamic behavior of a conformal field theory with the presence of diffusion, shear, and sound wave modes. There is also a family of purely damped electromagnetic modes which tend to the bosonic Matsubara frequencies in the long-wavelength regime.

Keywords: p-branes, AdS-CFT Correspondence, Classical Theories of Gravity, Black Holes.

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Contents

1. Introduction

1.1 Motivations and overview

Theoretical studies on black holes in asymptotically anti-de Sitter spacetimes have attracted substantial attention since the advent of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence [1 – 3]. In particular, the quasinormal-mode (QNM) spectra of various types of asymptotically AdS black holes have been analyzed since then (see refs. [4 – 30] for a sample). According to the AdS/CFT correspondence, an asymptotically AdS black hole is, in the CFT side, associated to a system in thermal equilibrium whose temperature is the Hawking temperature of the black hole. In such a context, blackhole perturbations correspond to small deviations from equilibrium of the CFT thermal system, and the characteristic damping time of perturbations, which is given by the inverse of the imaginary part of the fundamental QNM frequency, is a measure of the dynamical timescale of approach to thermal equilibrium of the corresponding conformal field theory [31].

The literature on QNM of AdS black holes includes studies taking into account a variety of different aspects such as the topology of the event horizon, the number of dimensions of the spacetime, the particular type of perturbation fields considered, and also the special parameters which characterize each different black hole itself. Each one of these variant properties reflects on the dual CFT. For instance, assuming the $(3+1)$-dimensional AdS spacetime contains a plane-symmetric black hole, then the holographic field theory is defined over the $(2+1)$-dimensional Minkowski spacetime, which is the conformal boundary of the bulk AdS spacetime. Moreover, different blackhole parameters characterize different dual plasmas in the CFT side, and different equilibrium states of such systems at the boundary.

An important issue in the study of the vibrational modes of black holes is the choice of appropriate boundary conditions. In the case of asymptotically flat spacetimes, the solutions to the wave equations governing linear perturbations are, near the boundaries, given by plane wave functions. QNM are then defined as solutions which satisfy physically well motivated boundary conditions, namely, purely ingoing waves at the horizon and purely outgoing waves at infinity (see refs. [32, 33] for reviews). For anti-de Sitter black holes, on the other hand, the condition at the future horizon is the same as for asymptotically flat spacetimes, but now there are no natural conditions to be imposed on the perturbation variables at the AdS infinity. These can be Dirichlet, Neumann, or Robin boundary conditions, depending on whether it is required that the field perturbations, their derivatives or a combination of both vanish at the AdS boundary, respectively. In the study of the evolution of a massless scalar field in $(3+1)$-, $(4+1)$-, and $(6+1)$-dimensional Schwarzschild-AdS spacetimes, Horowitz and Hubeny [31] computed the corresponding quasinormal-mode spectra by imposing Dirichlet boundary conditions on such a field at infinity. This option was well justified in that context, since by writing the radial part of the Klein-Gordon equation in a Schrödinger-like form, the resulting effective potential diverges at that boundary. The same boundary condition was used to study massless scalar and electromagnetic perturbations of $(2+1)$-dimensional Bañados-Teitelboim-Zanelli (BTZ) black holes [34]. For BTZ black holes, an analytical closed form for the quasinormal frequencies was derived [35], and it was verified that the quasinormal frequencies correspond exactly to the poles of retarded

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correlation functions in the dual $(1 + 1)$-dimensional CFT [36]. It was also suggested in ref. [37] that the relation between quasinormal modes and singularities of correlation functions should also hold for scalar fields in higher-dimensions, as far as the frequencies are computed by imposing Dirichlet boundary conditions on such fields at AdS infinity.

In the meantime, two fundamental difficulties arise when considering gravitational and/or electromagnetic perturbations of AdS black holes, particularly in higher dimensional spacetimes. The first problem is related to the arbitrariness in the choice of gauge-invariant perturbation fields. In fact, there is an infinity of gauge-invariant combinations of metric (or vector potential) fluctuations that can be used as fundamental variables governing the gravitational (or electromagnetic) perturbations. The second problem is related to the ambiguity in defining appropriate boundary conditions for the quasinormal modes. A traditional way to face such arbitrariness is opting for master variables that lead to equations generalizing those for perturbations in asymptotically flat spacetimes. That is to say, variables are chosen in such a way to put the radial part of the fundamental equations into a Schrödinger-like form. From now on, the corresponding master variables shall be called the Regge-Wheeler-Zerilli (RWZ) variables.[1] With such a choice of variables, it was investigated gravitational and/or electromagnetic perturbations of the Schwarzschild-AdS [38, 41 – 46], Reissner-Nordström-AdS [47 – 49], and Kerr-AdS [50] black holes, as well as the perturbations of black holes with non-spherical topologies [51, 52], including the plane-symmetric ones [53 – 55]. Analogously to the massless scalar field case, in all of these works the quasinormal modes were computed by imposing Dirichlet boundary conditions on the master fields at infinity. Alternative boundary conditions for the same Regge-Wheeler-Zerilli variables have been discussed in refs. [56, 57].

A different route was taken by Núñez and Starinets [58], who defined the quasinormal frequencies of a perturbation in an asymptotically AdS spacetime as "the locations in the complex frequency plane of the poles of the retarded correlator of the operators dual to that perturbation". To compute the real-time correlation functions, they suggested using the Lorentzian AdS/CFT prescription of refs. [37, 59]. The quasinormal-mode definition supplied by Núñez and Starinets was explored in ref. [60], where a new set of fundamental variables was introduced to study electromagnetic and gravitational perturbations of $(4 + 1)$-dimensional plane-symmetric black holes (or black branes, for short). It was shown there that the imposition of Dirichlet boundary conditions on such a new set of gauge-invariant variables at infinity leads exactly to the quasinormal frequencies associated to the corresponding black branes. In the present work these kind of fundamental variables shall be called the Kovtun-Starinets (KS) variables.

An important consequence of the Núñez-Starinets approach [58] is that the resulting quasinormal-mode spectra present a set of dispersion relations, here called hydrodynamic QNM, that behave like diffusion, shear, and sound wave modes in the long-wavelength, low-frequency limit [60]. These results are totally consistent with what is expected from the CFT point of view, and they provide a non-trivial test of the AdS/CFT correspondence. It

[1] In the first study of gravitational QNM in AdS spacetimes, Cardoso and Lemos [38] used the same kind of variables as the early works in asymptotically flat spacetimes by Regge and Wheeler [39], and by Zerilli [40].

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is also worth noticing that neither the electromagnetic diffusion mode nor the gravitational sound wave mode are obtained by imposing Dirichlet boundary conditions on the RWZ master variables. For Schwarzschild-AdS and topological-AdS $(3+1)$-dimensional black holes, it was only possible to obtain sound wave modes in the gravitational quasinormal spectra by requiring that a specific combination of the master field and its derivative vanishes at infinity [57, 61, 62].

1.2 The present work

1.2.1 General procedure

In this work the definition of QNM given by Núñez and Starinets [58] is applied to compute the quasinormal frequencies associated to electromagnetic and gravitational perturbations of $(3+1)$-dimensional plane-symmetric AdS black holes. The overall procedure is similar to that of ref. [60] and consists of the following steps:

(1) Initially the translation invariance of the static plane-symmetric AdS spacetimes is used to Fourier transform the fluctuation fields with respect to time and to the two Cartesian coordinates (x, y) of the plane.

(2) With the spatial wave vector chosen to be in the y-direction, both the electromagnetic and the gravitational perturbation fields are separated into two sets according to their behavior under the transformation $x \to -x$: odd (axial, or transverse), and even (polar, or longitudinal) perturbations.

(3) Each sector of perturbation fields is governed by a set of linearized differential equations. In all of the cases studied here, the complete set of perturbation equations can be decoupled in order to obtain a unique second-order differential equation, which is the fundamental equation of that perturbation sector. The fundamental equations are written in terms of gauge-invariant combinations of the perturbation fields, extending the original definitions of Kovtun-Starinets variables [60] to $(3+1)$-dimensional spacetimes.

(4) Then, the standard AdS/CFT prescription of ref. [37] is applied to express the real-time R-symmetry current and stress-energy tensor correlators in terms of quantities which represent the asymptotic behavior of perturbations near the AdS-space boundary. Such a procedure shows that the imposition of Dirichlet boundary conditions on Kovtun-Starinets variables at infinity leads to the poles of the CFT correlation functions, and therefore, according to the Núñez-Starinets definition of QNM, to the quasinormal spectra of the plane-symmetric AdS black holes.

(5) With well defined boundary conditions and a set of decoupled fundamental equations, the hydrodynamical limit of the QNM spectra is then analyzed. This limit is reached for perturbation modes in which the frequency and the wavenumber are much smaller than the Hawking temperature of the black hole.

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(6) The last step is numerically compute the electromagnetic and gravitational quasinormal dispersion relations for different blackhole parameters. For such a purpose, the Horowitz-Hubeny method [31], which reduces the problem of finding QNM frequencies to that of obtaining the roots of infinite polynomial equations, is used.

1.2.2 Main results

Among the new results found in the present work, it is worth mentioning the following ones.

- First, the derivation of the electromagnetic diffusion mode and the gravitational sound wave mode is performed by means of a traditional QNM calculation. These modes were earlier obtained by Herzog [63, 64], who utilized the AdS/CFT prescription [37] to directly compute the hydrodynamic limit of the CFT R-symmetry current and stress-energy tensor correlators.

- Second, it is found that the procedure of imposing Dirichlet boundary conditions on the gauge-invariant KS variables breaks the isospectrality between the axial and polar electromagnetic QNM, that follows from RWZ variables. As a result, the polar electromagnetic quasinormal modes are totally new, since the KS variable with Dirichlet boundary condition yields a different spectrum when compared to the RWZ variable with the same kind of boundary conditions. Regarding to the electromagnetic axial perturbations, the dispersion relations found here enlarge previous results of ref. [53].

- Third, it is found in addition that the complete spectra of electromagnetic QNM present a tower of purely damped modes which tend to Matsubara frequencies characteristic to bosonic systems in the long-wavelength regime. However, these quasinormal modes do not exist for all wavenumbers. In fact, there is a saturation value for the wavenumber above which the electromagnetic purely damped modes disappear.

- Fourth, another result to be mentioned is the difference between the spectrum of the gravitational polar perturbations, computed by using the KS variable, and that obtained using the RWZ master variable [54]. The differences are specially significant when the fluctuation wavenumber is of the same order of the magnitude of the blackhole temperature.

- And last but not least, the dispersion relations calculated here complete the previous results for axial gravitational QNM of (3+1)-dimensional plane-symmetric AdS black holes [53, 54].

1.2.3 Structure of the paper

The layout of the present article is as follows. Section 2 contains a brief summary of the relation between the plane-symmetric AdS_4 black holes and the eleven-dimensional supergravity solution associated with a stack of N M2-branes. In the sequence a detailed study of the electromagnetic quasinormal modes is performed (section 3): The basic equations are obtained in section 3.1 and the connection between the blackhole perturbations and the

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CFT R-symmetry currents is explored in section 3.2; the hydrodynamic modes of the electromagnetic perturbations are studied in section 3.4, and the general dispersion relations of electromagnetic QNM are reported in section 3.5. The gravitational quasinormal modes are studied in section 4: The basic equations are obtained in section 4.1; 4.2 is devoted to investigate the relation between the gravitational QNM and the stress-energy tensor correlators in the holographic CFT; the hydrodynamic modes of the gravitational perturbations are studied in section 4.4, and the numerical results for the dispersion relations of the remaining gravitational QNM are presented in section 4.5. The article is completed, in section 5, with the analysis and interpretation of the main results.

1.3 Notation and conventions

Natural units are going to be used throughout this paper, i.e., the speed of light c, Boltzmann constant k_B, and Planck constant $\hbar$ are all set to unity, $c = k_B = \hbar = 1$. Regarding to notation, capital Latin indices M, N, ... vary over the coordinates of the whole AdS spacetime, while Greek indices μ, ν, ... label different coordinates at the boundary, and small Latin indices i, j, ... vary only over the spacelike coordinates at the boundary. The convention for the metric signature and for all the definitions of curvature tensors follow ref. [65].

2. M2-branes and the plane-symmetric black holes

2.1 The background spacetime

Since the QNM definition of Núñez-Starinets [58], that is adopted in this work, makes heavy use of the relation between AdS black holes and conformal field theories at finite temperature, it becomes important to review here how the plane-symmetric AdS_4 black holes arise in the context of the AdS/CFT conjecture.[2]

A fundamental role in the AdS/CFT correspondence[3] is played by extended two-dimensional objects known as M2-branes [68]. The world-volume theory of N M2-branes is a $(2+1)$-dimensional non-Abelian Yang-Mills theory which presents $\mathcal{N} = 8$ supersymmetries in addition to a SU(N) gauge group. The coupling constant of the theory flows to strong coupling in the infrared limit, and it is believed that the flow is to an infrared-stable fixed point that describes a superconformal field theory [69]. This CFT also has an emerging R-charge symmetry which is expanded to SO(8).

From the supergravity point of view, a stack of N M2-branes is described by a nonextremal solution to the supergravity equations of motion, characterized by the metric [63, 70, 71]

$$ds^2 = H^{-2/3}(\widetilde{r}) \left[-\mathfrak{h}(\widetilde{r})dt^2 + dx^2 + dy^2\right] + H^{1/3}(\widetilde{r}) \left[\mathfrak{h}^{-1}(\widetilde{r})d\widetilde{r}^{\,2} + \widetilde{r}^{\,2} d\Omega_7^2\right], \tag{2.1}$$

where

$$H(\widetilde{r}) = 1 + \left(\frac{R}{\widetilde{r}}\right)^6 \qquad \text{and} \qquad \mathfrak{h}(\widetilde{r}) = 1 - \left(\frac{\widetilde{r}_0}{\widetilde{r}}\right)^6, \tag{2.2}$$

[2]The brief summary presented in this section is based on material found in refs. [63, 66, 67].
[3]The interest here is the AdS_4/CFT_3 correspondence.

and by a four-form field whose dual Hodge is given by

$$\star F_4 = F_7 = 6R^6 \mathrm{Vol}(S^7)\boldsymbol{\varepsilon}, \tag{2.3}$$

where $\boldsymbol{\varepsilon}$ stands for the Levi-Civita tensor on S^7. According to the AdS/CFT correspondence [1–3], the $(2+1)$-dimensional $\mathcal{N}=8$ CFT is dual to M-theory on the background spacetime (2.1). Furthermore, the quantization condition on the F_4 flux connects the parameter R to the number of branes N [72]:

$$R^9\pi^5 = N^{3/2}\kappa_{11}^2\sqrt{2}, \tag{2.4}$$

where κ_{11} is the gravitational coupling strength in $(10+1)$-dimensional supergravity.

In the large N limit ($N \gg 1$), one can consider only the near-horizon region ($\widetilde{r} \ll R$) of the spacetime (2.1). Function $H(\widetilde{r})$ then reduces to $H(\widetilde{r}) = R^6/\widetilde{r}^6$. Moreover, defining a new radial coordinate by $r = \widetilde{r}^2/2R$, metric (2.1) becomes

$$ds^2 = \frac{4r^2}{R^2}\left[-\mathfrak{h}(r)dt^2 + dx^2 + dy^2\right] + \frac{R^2}{4r^2}\frac{dr^2}{\mathfrak{h}(r)} + R^2 d\Omega_7^2. \tag{2.5}$$

The AdS part of the metric (2.5), associated to the coordinates $\{t, x, y, r\}$, is identical to the solution of Einstein equations with negative cosmological term corresponding to a $(3+1)$-dimensional plane-symmetric AdS black hole [73–76]:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + \frac{r^2}{L^2}(dx^2+dy^2), \tag{2.6}$$

where the horizon function $f(r)$ is given by

$$f(r) = \left(\frac{r}{L}\right)^2 \mathfrak{h}(r) = \left(\frac{r}{L}\right)^2\left(1 - \frac{r_0^3}{r^3}\right), \tag{2.7}$$

and the seven-sphere radius R has been rewritten as $R = 2L$, with L now representing the AdS radius of the spacetime (2.6). Parameters r_0 and L are related to the blackhole Hawking temperature T by

$$T = \frac{3}{4\pi}\frac{r_0}{L^2}. \tag{2.8}$$

2.2 Normalization of the field action

The full theory is the eleven-dimensional supergravity on $\mathrm{AdS}_4 \times S^7$, and the existence of a compact seven-sphere enables one to consistently reduce the theory to Einstein-Maxwell theory on AdS_4 [67, 77, 78]. The main objective in summarizing such a procedure here is to make explicit the dependence of the action for the fields in the AdS_4 spacetime on the number of colours N, which is one of the parameters characterizing the holographic CFT.

Upon Kaluza-Klein dimensional reduction, the Maxwell gauge field A_M arises from a combination of metric and F_4 form perturbations in the eleven-dimensional supergravity. This field corresponds to a U(1) subgroup of the SO(8) symmetry group of the complete spacetime (2.1). The mechanism of dimensional reduction also furnishes the $(3+1)$-dimensional Einstein-Maxwell action with a negative cosmological constant $\Lambda = -3/L^2$:

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left(\mathcal{R} + \frac{6}{L^2} - L^2 F_{MN}F^{MN}\right), \tag{2.9}$$

where $\mathcal{R}$ denotes the Ricci scalar and F_{MN} is the electromagnetic strength tensor, and for the purposes of the present analysis the electromagnetic Lagrangian $\mathcal{L}_{\rm em} \sim F_{MN}F^{MN}$ is considered as a perturbation on the gravitational Lagrangian $\mathcal{L}_{gr} \sim \mathcal{R}+6/L^2$. It is assumed that the gravitational coupling constants in four and eleven dimensions are related by means of the seven-sphere volume [67],

$$\frac{1}{2\kappa_4^2} = \frac{R^7 {\rm Vol}(S^7)}{2\kappa_{11}^2}. \tag{2.10}$$

Then, considering that the volume of a unitary seven-sphere is ${\rm Vol}(S^7) = \pi^4/3$, and using the standard normalization (2.4) for κ_{11}, it is found

$$\frac{1}{2\kappa_4^2} = \frac{\sqrt{2}N^{3/2}}{24\pi L^2}. \tag{2.11}$$

Action (2.9) with the gravitational constant κ_4 given in terms of the number of colours N and of the anti-de Sitter radius L is the desired result, which is needed for the development of the present work.

3. Electromagnetic quasinormal modes

3.1 Perturbation equations

In the AdS/CFT context, the electromagnetic field in the AdS bulk couples to the CFT R-symmetry currents at the spacetime boundary. Hence, in order to construct the current-current two-point correlation functions in the CFT, it is necessary to consider fluctuations of the gauge field A_M. Such a field is implicitly defined by

$$F_{MN} = \partial_M A_N - \partial_N A_M, \tag{3.1}$$

with F_{MN} satisfying equations of motion derived from the action (2.9). Therefore, considering the electromagnetic field as a perturbation on the background spacetime of metric (2.6), the resulting equations of motion for A_M are the usual Maxwell equations

$$\partial_M \left(\sqrt{-g} g^{MA} g^{NB} F_{AB}\right) = 0, \tag{3.2}$$

where g_{MN} stands for the metric components given by (2.6).

When looking for solutions to eqs. (3.2), by taking into account the isometries of the background metric (2.6), it is convenient to decompose the gauge field in terms of Fourier transforms as follows

$$A_M(t,x,y,r) = \frac{1}{(2\pi)^3}\int d\omega\, dk_x\, dk_y\, e^{-i\omega t + ik_x x + ik_y y}\widetilde{A}_M(\omega, k_x, k_y, r). \tag{3.3}$$

Furthermore, without loss of generality, in the plane-symmetric background spacetime (2.6) one may choose the wave three-vector k in the form $k_\mu = (k_0, k_x, k_y) = (-\omega, 0, q)$. This is carried out through an appropriate rotation in the $x-y$ plane, in such a way that the Fourier modes of the gauge field propagate along the y direction only. With such a choice, the electromagnetic perturbations A_M can be split into two independent sets according to their behavior under parity operation, $x \to -x$:

- Axial (odd, or transverse) perturbations: A_x;
- Polar (even, or longitudinal) perturbations: A_t, A_y, A_r.

Since these two sets of perturbations are orthogonal sets, they can be studied separately, as it is done in the following.

3.1.1 Equations for axial perturbations

Axial electromagnetic perturbations are governed by the transverse component of Maxwell equations (3.2), which gives

$$f\frac{d^2A_x}{dr^2}+\frac{df}{dr}\frac{dA_x}{dr}+\left(\frac{\omega^2r^2-q^2L^2f}{fr^2}\right)A_x=0, \tag{3.4}$$

where, to simplify notation, the tilde was dropped, $\widetilde{A}_x\to A_x$. Moreover, it follows from eqs. (3.1) and (3.3), together with $k_\mu=(-\omega,0,q)$, that A_x is proportional to the transverse component of the electric field: $E_x=i\omega A_x$. Therefore, being a gauge-invariant quantity, A_x is also a good candidate as master variable for axial perturbations. In fact, it is possible to cast eq. (3.4) into a Schrödinger-like form [53]

$$\left(\frac{d^2}{dr_*^2}+\omega^2\right)\Psi^{(-)}=f\left(\frac{qL}{r}\right)^2\Psi^{(-)}, \tag{3.5}$$

where $\Psi^{(-)}(r)=A_x(r)$, and the tortoise coordinate r_* is defined in terms of the radial coordinate r by

$$\frac{dr}{dr_*}=f(r). \tag{3.6}$$

For the present purposes it is convenient to change coordinates to the inverse radius $u=r_0/r$, and then, by writing eq. (3.5) in terms of E_x it results

$$E_x^{''}+\frac{\mathfrak{h}^{'}}{\mathfrak{h}}E_x^{'}+\frac{\mathfrak{w}^2-\mathfrak{q}^2\mathfrak{h}}{\mathfrak{h}^2}E_x=0, \tag{3.7}$$

where the primes denote derivation with respect to the variable u, and $\mathfrak{w}$ and $\mathfrak{q}$ are respectively the normalized frequency and wavenumber, defined by

$$\mathfrak{w}=\frac{3\omega}{4\pi T}\qquad\text{and}\qquad\mathfrak{q}=\frac{3q}{4\pi T}, \tag{3.8}$$

with T being the Hawking temperature of the black hole, given by eq. (2.8). Function $\mathfrak{h}$ is obtained from eqs. (2.2), or from eq. (2.7), and in terms of the variable $u=r_0/r$ reads

$$\mathfrak{h}\equiv\mathfrak{h}(u)=1-u^3\,. \tag{3.9}$$

Quantity E_x shall be the fundamental gauge-invariant variable to be used in the present analysis of the QNM modes for axial electromagnetic perturbations.

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3.1.2 Equations for polar perturbations

Differently from the axial electromagnetic perturbations, the components of the gauge field A_M corresponding to the set of polar perturbations are not gauge invariant. The gauge freedom can then be used in order to simplify the relevant equations of motion. In fact, the invariance of Maxwell equations under the gauge transformation $A_M \to A_M + \partial_M \lambda$ allows choosing λ in such a way that one of the components A_t, A_y, or A_r vanishes. For instance, it is possible to work in the so-called radial gauge, in which $A_r = 0$ [63]. In this gauge, Maxwell equations (3.2) corresponding to the polar perturbations are

$$\omega r^2 \frac{d}{dr} A_t + q L^2 f \frac{d}{dr} A_y = 0, \tag{3.10}$$

$$r^2 \frac{d^2}{dr^2} A_t + 2r \frac{d}{dr} A_t - \frac{L^2}{f} \left(q\omega A_y + q^2 A_t \right) = 0, \tag{3.11}$$

$$f \frac{d^2}{dr^2} A_y + \frac{df}{dr}\frac{d}{dr} A_y + \frac{1}{f} \left(\omega q A_t + \omega^2 A_y \right) = 0. \tag{3.12}$$

Notice that this set of equations does not constitute a linearly independent system, and any subset composed by two of such equations determines the two remaining unknown components of the gauge field, A_t and A_y.

From now on, A_t or A_y could be adopted as a primary variable and equations (3.10)–(3.12) could be decoupled in order to find a unique differential equation for one of these functions. However, to avoid the inconvenience of dealing with gauge dependent quantities, it is interesting to use the electric field components, which are gauge-invariant quantities. Even though this choice eliminates gauge-dependent potential fields, a residual ambiguity is left: from the electric field components $E_r = dA_t/dr$ and $E_y = i(qA_t + \omega A_y)$, what is the best choice?

The answer to the last question is not simple and both of the possible answers have been tried in the literature. For instance, inspired by preceding works studying blackhole perturbations in asymptotically flat spacetimes [79], Cardoso and Lemos opted for the radial component E_r in studying QNM of Schwarzschild-AdS black holes [38], and plane-symmetric AdS black holes [53]. Introducing a new variable $\Psi^{(+)}(r) = r^2 E_r(r)$ and using eqs. (3.10) and (3.11), they were able to reduce the system of equations into a unique ordinary differential equation of Schrödinger type

$$\left(\frac{d^2}{dr_*^2} + \omega^2 \right) \Psi^{(+)} = f \left(\frac{qL}{r} \right)^2 \Psi^{(+)}, \tag{3.13}$$

which has the same form as the fundamental equation for axial perturbations, eq. (3.5). An interesting consequence of this fact is that the QNM spectra for both the axial and polar perturbations, with the same boundary conditions, are identical [38, 53]. As a matter of fact, an open question left behind in the early works computing electromagnetic QNM of AdS black holes is the lack of a convincing physical justification for the choice of Dirichlet boundary conditions at infinity for both the polar and the axial perturbations. Such an issue will be considered in the sequence of this work.

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As far as one is interested in computing the polar electromagnetic quasinormal frequencies, it will be shown in section 3.2 that E_y is more appropriate as a fundamental variable than E_r. This was the choice made, for instance, by Kovtun and Starinets in studying QNM of black branes in $(4+1)$-dimensional spacetimes [60]. Following these authors, in the present work E_y is adopted as the fundamental variable to be used to determine the QNM spectrum of polar electromagnetic perturbations. Having made this choice, one then writes equations in terms of the independent variable $u = r_0/r$. With this, eqs. (3.10) and (3.11) written for E_y lead to

$$E_y'' + \frac{\mathfrak{w}^2 \mathfrak{h}'}{\mathfrak{h}\left(\mathfrak{w}^2 - \mathfrak{q}^2\mathfrak{h}\right)} E_y' + \frac{\left(\mathfrak{w}^2 - \mathfrak{q}^2\mathfrak{h}\right)}{\mathfrak{h}^2} E_y = 0, \tag{3.14}$$

where $\mathfrak{w}$ and $\mathfrak{q}$ are respectively the normalized frequency and wavenumber, defined by eqs. (3.8), and $\mathfrak{h}$ is given by eq. (3.9).

According to the Núñez-Starinets QNM definition [58], once one has found the fundamental perturbation equations for a field in the AdS spacetime, the next step is establishing explicit relations between the perturbation variables and the corresponding retarded Green functions in the holographic CFT. It is exactly from these relations that will emerge the boundary conditions to be imposed on the perturbation fields at infinity, viz, the conditions that lead to the singularities of the two-point correlation functions in the boundary field theory, and consequently to the quasinormal frequencies of the fluctuation modes. Such a task is performed in what follows.

3.2 R-current correlation functions

In the case of electromagnetic perturbations, the AdS/CFT correspondence [1–3] tells that, in the strong coupling, large N limit, the information on the thermal correlation functions of the R-symmetry currents are encoded into the electric field components E_j $(j = x, y)$, which are solutions to the differential equations (3.7) and (3.14), respectively. It can be shown that, close to the horizon $(u \approx 1)$, such functions are given approximately by $E_j = \mathfrak{h}^{\pm i\mathfrak{w}/3}$, where the negative (positive) exponent corresponds to ingoing (outgoing) waves. Moreover, depending on the sign of the exponent, the boundary values of the perturbation functions act as sources of retarded or advanced Green functions in the dual CFT. To compute the retarded two-point functions, one has to opt for the negative exponent. It is also necessary to know the asymptotic form of the perturbation functions close to the infinite boundary $(u \approx 0)$. A simple analysis shows that the solutions of equations (3.7) and (3.14) which satisfy incoming-wave condition at horizon present the following behavior around $u = 0$:

$$E_x = \mathcal{A}_{(x)}(\mathfrak{w}, \mathfrak{q}) + \cdots + \mathcal{B}_{(x)}(\mathfrak{w}, \mathfrak{q})u + \cdots, \tag{3.15}$$

$$E_y = \mathcal{A}_{(y)}(\mathfrak{w}, \mathfrak{q}) + \cdots + \mathcal{B}_{(y)}(\mathfrak{w}, \mathfrak{q})u + \cdots, \tag{3.16}$$

where ellipses denote higher powers of u for each one of the independent solutions. Symbols $\mathcal{A}_{(j)}(\mathfrak{w}, \mathfrak{q})$ and $\mathcal{B}_{(j)}(\mathfrak{w}, \mathfrak{q})$, introduced in the above equations, stand for the connection coefficients associated to the corresponding differential equations for E_x and E_y.

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To proceed further and calculate the correlation functions, the electromagnetic action at the boundary needs to be determined. It is usual to split the action as $S_{EM} = S_{horizon} + S_{boundary}$. Using the equations of motion and the preceding definitions it follows

$$S_{boundary} = \frac{\chi}{2}\lim_{u\to 0}\int \frac{d\mathfrak{w}d\mathfrak{q}}{(2\pi)^2}\left[\frac{\mathfrak{h}}{\mathfrak{w}^2-\mathfrak{q}^2\mathfrak{h}}E_y^{'}(u,k)E_y(u,-k)+\frac{\mathfrak{h}}{\mathfrak{w}^2}E_x^{'}(u,k)E_x(u,-k)\right], \tag{3.17}$$

where

$$\chi = \frac{8\pi T L^2}{3\kappa_4^2} = \frac{(2N)^{3/2}T}{9} \tag{3.18}$$

is the electric susceptibility of the dual system [63, 67].

In order to apply the Lorentzian AdS/CFT prescription of ref. [37], the asymptotic solutions (3.15) and (3.16) are used to write the derivatives of the electric field in terms of the boundary values of the three-vector potential $A_\mu^0(k) = A_\mu(u\to 0, k)$. The R-current correlation functions $C_{\mu\nu}$ are proportional to the coefficients of the terms containing the product $A_\mu^0(k)A_\nu^0(-k)$ that appears in the action (3.17). It is then found:

$$\begin{aligned} C_{tt} &= \chi\frac{\mathfrak{q}^2}{(\mathfrak{w}^2-\mathfrak{q}^2)}\frac{\mathcal{B}_{(y)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(y)}(\mathfrak{w},\mathfrak{q})}, \qquad C_{yy} = \chi\frac{\mathfrak{w}^2}{(\mathfrak{w}^2-\mathfrak{q}^2)}\frac{\mathcal{B}_{(y)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(y)}(\mathfrak{w},\mathfrak{q})}, \\ C_{ty} &= -\chi\frac{\mathfrak{w}\mathfrak{q}}{(\mathfrak{w}^2-\mathfrak{q}^2)}\frac{\mathcal{B}_{(y)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(y)}(\mathfrak{w},\mathfrak{q})}, \quad C_{xx} = \chi\frac{\mathcal{B}_{(x)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(x)}(\mathfrak{w},\mathfrak{q})}. \end{aligned} \tag{3.19}$$

Moreover, for a $(2+1)$-dimensional CFT at finite temperature, the current-current correlation functions can be written in terms of the transverse and longitudinal self-energies $\Pi^T(\mathfrak{w},\mathfrak{q})$ and $\Pi^L(\mathfrak{w},\mathfrak{q})$, respectively (See appendix A for a summary of such relations). Hence, comparing eqs. (3.19) to eqs. (A.1)–(A.4) of appendix A, one finds

$$\Pi^T(\mathfrak{w},\mathfrak{q}) = \chi\frac{\mathcal{B}_{(x)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(x)}(\mathfrak{w},\mathfrak{q})}, \qquad \Pi^L(\mathfrak{w},\mathfrak{q}) = \chi\frac{\mathcal{B}_{(y)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(y)}\mathfrak{w},\mathfrak{q})}. \tag{3.20}$$

These results show that the retarded two-point correlation functions are fully determined by the ratio between the connection coefficients of equations (3.7) and (3.14). Furthermore, the poles of the thermal correlation functions are given by the zeros of the coefficients $\mathcal{A}_{(x)}(\mathfrak{w},\mathfrak{q})$ and $\mathcal{A}_{(y)}(\mathfrak{w},\mathfrak{q})$. According to ref. [58], the poles of $C_{\mu\nu}$ define the electromagnetic QNM frequencies of the black hole localized in the AdS spacetime. Such frequencies are then obtained by imposing Dirichlet boundary conditions on the electric field components E_x and E_y at $u = 0$, with E_x and E_y being functions that satisfy also an incoming-wave condition at the horizon.

3.3 QNM and the gauge-invariant variables

At this stage one could ask whether imposing Dirichlet conditions at the boundary ($u = 0$) on the Regge-Wheeler-Zerilli (RWZ) variables $\Psi^{(\pm)}$ would produce the same QNM spectra as the spectra obtained by imposing the same boundary conditions onto the Kovtun-Starinets (KS) variables $E_{x,y}$. For the transverse electromagnetic sector, the answer to this question is quite easy to find. In fact, variables $\Psi^{(-)}$ and E_x are proportional to each other, so that both of the obtained QNM spectra, either using the KS variable or using the RWZ variable, are identical. In the case of polar electromagnetic perturbations, equations for the KS variable E_y and for the RWZ variable $\Psi^{(+)}$ do not have the same form and, in addition, E_y and $\Psi^{(+)}$ are independent variables, so that the answer is not immediate. In fact, as it is shown below (see sections 3.4 and 3.5), the QNM spectrum obtained from E_y is different from the QNM spectrum obtained from $\Psi^{(+)}$.

3.4 Dispersion relations for the hydrodynamic QNM

The hydrodynamic limit of perturbations corresponds to the small frequency ($\mathfrak{w} \ll 1$) and small wavenumber ($\mathfrak{q} \ll 1$) region of the spectrum of the respective Fourier modes. In general, the quasinormal modes can be classified according the behavior of the dispersion relations in the hydrodynamic limit, and in this respect there are two classes. There is a set of QNM for which the frequency $\mathfrak{w}(\mathfrak{q})$ vanishes when $\mathfrak{q} \to 0$. Such modes are named here hydrodynamic quasinormal modes. But there is another kind of QNM for which the corresponding frequency in the long-wavelength limit is nonzero. To distinguish these two kind of modes from each other, the modes belonging to the later kind are denominated non-hydrodynamic quasinormal modes. In this section, the dispersion relations of the electromagnetic hydrodynamic QNM are studied by means of analytical and numerical methods. The electromagnetic non-hydrodynamic QNM shall be object of study in the next section.

From the CFT point of view, it is expected that at least one of the electromagnetic QNM should show the typical behavior of a diffusion mode in the hydrodynamic limit. Trying to find such a mode, one then looks for solutions to eqs. (3.7) and (3.14) in the form of power series in $\mathfrak{w}$ and $\mathfrak{q}$, under the assumption $\mathfrak{w} \sim \mathfrak{q}$. Written in terms of the variables $F_j = \mathfrak{h}^{i\mathfrak{w}/3} E_j$ (for $j = x, y$), which are more appropriate for the present analysis, perturbation equations (3.7) and (3.14) may be cast as

$$F_j'' + \frac{u^2}{\mathfrak{h}} \left(2i\mathfrak{w} - 3a_j\right) F_j' + \frac{1}{\mathfrak{h}^2} \left[i\mathfrak{w}(2u + u^4 - 3a_j u^4) + \mathfrak{w}^2(1 - u^4) - \mathfrak{q}^2\mathfrak{h}\right] F_j = 0, \tag{3.21}$$

where $a_x = 1$ and $a_y = \mathfrak{w}^2/(\mathfrak{w}^2 - \mathfrak{q}^2\mathfrak{h})$. After relabelling parameters as $\mathfrak{w} \to \lambda\mathfrak{w}$ and $\mathfrak{q} \to \lambda\mathfrak{q}$ with $\lambda \ll 1$, it is assumed that solutions of eqs. (3.21) can be expanded in the form

$$F_j(u) = F_j^0(u) + \lambda F_j^1(u) + \lambda^2 F_j^2(u) + \cdots, \tag{3.22}$$

where the coefficients $F_j^\alpha(u)$, with $\alpha = 0, 1, 2, \ldots$, represent arbitrary functions of variable u, and which are also homogeneous functions of degree α on $\mathfrak{w}$ and $\mathfrak{q}$.

The boundary condition of being ingoing waves at the horizon imposed on E_j, when translated to the new functions F_j, implies their dominant terms in expansion (3.22) must assume constant values close the horizon ($u \approx 1$). Then, in terms of the expansion (3.22) one has the following conditions

$$F_j^0(1) = \text{constant}, \qquad F_j^1(1) = F_j^2(1) = \cdots = 0. \tag{3.23}$$

It is now possible to solve eqs. (3.21) order by order and, after imposing the boundary conditions given in eqs. (3.23), the following expansions are found:

$$E_x = C_x \mathfrak{h}^{-i\mathfrak{w}/3} \left[1 - i\mathfrak{w}\frac{\sqrt{3}}{3}\left(\frac{\pi}{3} - \arctan\frac{1+2u}{\sqrt{3}}\right) + \frac{i\mathfrak{w}}{2}\ln\frac{1+u+u^2}{3} + \mathcal{O}(\mathfrak{w}^2)\right], \tag{3.24}$$

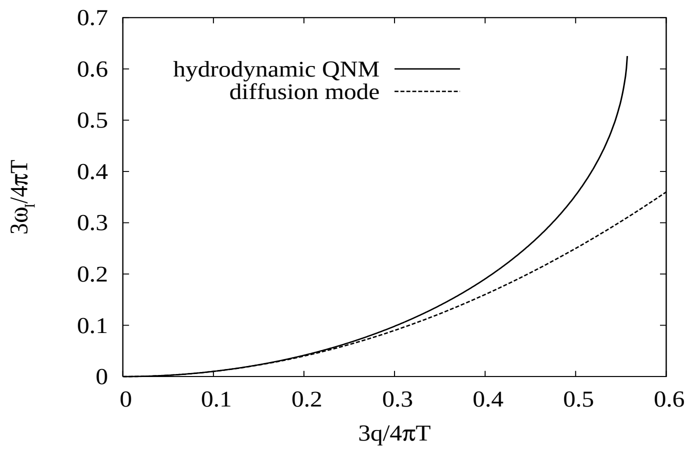

Figure 1: The dispersion relation for the only electromagnetic hydrodynamic QNM (solid line), which is purely damped, $\mathfrak{w} = -i\mathfrak{w}_I$, and corresponds to a polar perturbation. The dotted line is the diffusion mode $\mathfrak{w}_I = \mathfrak{q}^2$, which approaches the quasinormal frequency in the hydrodynamic limit $\mathfrak{w}, \mathfrak{q} \ll 1$.

$$E_y = C_y \mathfrak{h}^{-i\mathfrak{w}/3} \left[1 + \frac{i\mathfrak{q}^2}{\mathfrak{w}}(1-u) - i\mathfrak{w}\frac{\sqrt{3}}{3}\left(\frac{\pi}{3} - \arctan\frac{1+2u}{\sqrt{3}}\right) + \frac{i\mathfrak{w}}{2}\ln\frac{1+u+u^2}{3} + \mathcal{O}(\mathfrak{w}^2)\right], \quad (3.25)$$

where C_x and C_y are arbitrary normalization constants. One finds from eq. (3.24) no solution satisfying the Dirichlet condition at the AdS spacetime boundary, $E_x(0) = 0$, and which is at the same time compatible with the hydrodynamic approximation $\mathfrak{w}, \mathfrak{q} \ll 1$. This means there is no axial electromagnetic hydrodynamic QNM, and no R-charge diffusion in the transverse direction to the spatial wave vector, as expected from the CFT point of view. On the other hand, the condition $E_y(0) = 0$ and eq. (3.25) lead to[4]

$$\mathfrak{w} = -i\mathfrak{q}^2 \qquad \Longrightarrow \qquad \omega = -\frac{3i}{4\pi T}q^2, \quad (3.26)$$

from where one can read the diffusion coefficient $D = 3/4\pi T$. It is worth noticing that this diffusion mode is not found if one uses the RWZ master variable $\Psi^{(+)}$ instead of E_y.

As seen above, the hydrodynamic limit of perturbation equations comprises a very special interval in the space of parameters $\mathfrak{w}$ and $\mathfrak{q}$. Besides the physical relevance of this regime, it corresponds to the very rare situations where analytical expressions can be found for the quasinormal frequencies. In the great majority of cases, numerical methods have to be employed in order to find the complete dispersion relations $\mathfrak{w} \times \mathfrak{q}$. In the sequence, the Horowitz-Hubeny method [31] is used to compute the dispersion relation $\mathfrak{w} = -i\mathfrak{w}_I(\mathfrak{q})$ for the electromagnetic hydrodynamic QNM, which in the limit of small wavenumbers corresponds to the diffusion mode found above. The result is shown in figure 1. One sees the deviation of the exact dispersion relation curve (solid line) from the hydrodynamic limit curve $\mathfrak{w}_I = \mathfrak{q}^2$ (dotted line). Another interesting fact is that the quasinormal frequency $\mathfrak{w}(\mathfrak{q})$ disappears for $\mathfrak{q}$ larger than approximately 0.557. This is characteristic to all the electromagnetic purely damped modes, as analyzed in the next section (see table 1).

[4]This result was also found through direct calculation of the hydrodynamic limit of correlation functions by Herzog [63, 64]. The comparison to that result is in fact a test for the analysis performed in section 3.2.

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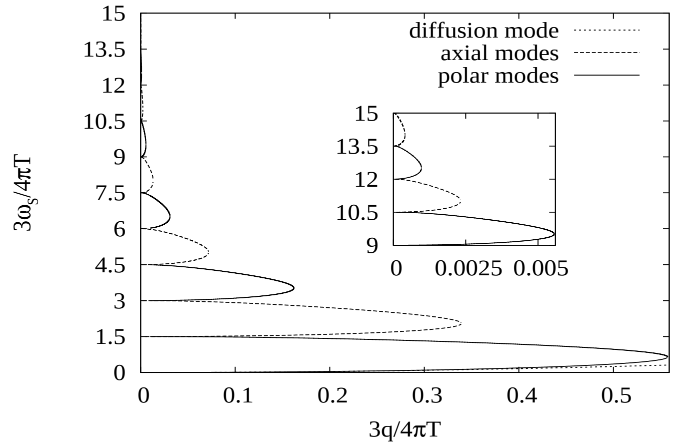

Figure 2: The dispersion relations for polar (solid lines) and axial (dashed lines) purely damped electromagnetic QNM. The dotted (lowest) line is the diffusion mode $\mathfrak{w}_s = \mathfrak{q}^2$, which approaches the $n_s = 0$ quasinormal frequency in the hydrodynamic limit $\mathfrak{w}, \mathfrak{q} \ll 1$. The insert shows the behavior of the dispersion relations for higher quasinormal frequencies.

3.5 Dispersion relations for the non-hydrodynamic QNM

As mentioned earlier, one of the goals of the present analysis is to obtain the electromagnetic quasinormal modes of plane-symmetric AdS_4 spacetimes and to compare the present results with the results of ref. [53]. As verified in the hydrodynamic limit (section 3.4), the QNM spectra calculated here may be quite different from the spectra obtained in that work because of the use of different fundamental variables, and therefore a more careful study on the dispersion relations of these modes is justified.

3.5.1 Purely damped modes

For small values of $\mathfrak{q}$, electromagnetic perturbations of AdS black holes present a special set of modes which are purely damped. These are not usual QNM since the real part of the frequencies vanishes eliminating the oscillatory behavior of the perturbations which is characteristic of QNM. Furthermore, the frequencies $\mathfrak{w}(\mathfrak{q})$ of such modes cannot be, in general, associated to hydrodynamic poles since most of the purely damped modes have nonvanishing frequencies in limit as the wavenumber $\mathfrak{q}$ goes to zero. To distinguish from the regular QNM, the purely imaginary frequencies of both the axial and the polar perturbations shall be labelled by a special quantum number, n_s, that assumes just integer values, starting by $n_s = 0$ for the hydrodynamic diffusion mode. As it was shown above, the axial sector of electromagnetic perturbations does not present quasinormal modes in the hydrodynamic limit, so that the set of axial purely damped modes starts at $n_s = 1$.

An interesting property of electromagnetic QNM of AdS black holes in $(3 + 1)$-dimensional spacetimes has been recently discovered by Herzog and collaborators [67]: The current-current correlators are analytical functions at $\mathfrak{q} = 0$, meaning that there are no quasinormal frequencies for null wavenumber. It can be shown that such a property is a consequence of the well known duality relation between electric and magnetic fields in vacuum. In fact, using the invariance of Maxwell equations under the duality operation,

Polar			Axial		
n_s	$\mathfrak{q}_{\rm lim} \times 10^3$	$\mathfrak{w}_{\rm lim}$ (interval)	n_s	$\mathfrak{q}_{\rm lim} \times 10^3$	$\mathfrak{w}_{\rm lim}$ (interval)
(0,1)	557.319	[0.648111, 0.648429]	(1,2)	339.330	[2.04771, 2.04811]
(2,3)	162.034	[3.52507, 3.52562]	(3,4)	71.8726	[5.01701, 5.01788]
(4,5)	31.0102	[6.51286, 6.51384]	(5,6)	13.1892	[8.00994, 8.01169]
(6,7)	5.55917	[9.50785, 9.51033]	(7,8)	2.32839	[11.0057, 11.0100]
(8,9)	0.970660	[12.5041, 12.5097]	(9,10)	0.403180	[14.0009, 14.0114]

Table 1: Approximate limiting values of frequencies $\mathfrak{w} = -i\mathfrak{w}_{\rm lim}$ and wavenumbers $\mathfrak{q}_{\rm lim}$ for purely damped electromagnetic modes. The brackets indicate that the actual limiting values lie between the two indicated endpoints in each case.

electric field $\leftrightarrow$ magnetic field, and the invariance of the correlation functions under rotations in the case of null wavenumber (zero momentum), it was shown that the transverse and longitudinal self-energies, $\Pi^T(\mathfrak{w}, 0)$ and $\Pi^L(\mathfrak{w}, 0)$, are well behaved functions of the frequency for all values of $\mathfrak{w}$.

On the other hand, as verified through the numerical results for purely damped modes, there are quasinormal frequencies even for wavenumbers very close to zero. In fact, as shown in figure 2, the small wavenumber limit ($\mathfrak{q} = \epsilon$, with ϵ very small but non-zero[5]) of the corresponding purely imaginary quasinormal frequencies, $\mathfrak{w} = -i\mathfrak{w}_s$, is given approximately by

$$\mathfrak{w}_s = \frac{3}{2} n_s \qquad \Longrightarrow \qquad \omega_s = \omega_{n_s} \equiv 2\pi T n_s, \tag{3.27}$$

where ω_{n_s} are the Matsubara frequencies of a generic quantum bosonic system. Moreover, as it is also seen from figure 2, the hydrodynamic pole $n_s = 0$ is similar to other purely damped modes. As a matter of fact, the only property that distinguishes a particular purely damped mode from another is the behavior of these modes around $\mathfrak{q} = 0$: The only QNM satisfying the condition $\lim_{\mathfrak{q}\to 0} \mathfrak{w}(\mathfrak{q}) = 0$ is the hydrodynamic mode.

At the opposite side of the quasinormal spectrum, i.e., for larger values of $\mathfrak{q}$, there are saturation points at a maximum wavenumber value, $\mathfrak{q}_{\rm lim}$, beyond which the specific mode disappears (See, however, figures 4 and 5). This seems to happen for everyone of the modes, with $\mathfrak{q}_{\rm lim}$ decreasing for higher overtones (cf. figure 2). The curves representing dispersion relations associated to two different but contiguous modes meet each other exactly at the saturation point, i.e., the two dispersion relation curves coincide at that point. The axial modes group in pairs according to the relation $n_s = \{(1, 2), (3, 4), (5, 6), \ldots\}$, while the polar modes are paired as $n_s = \{(0, 1), (2, 3), (4, 5), \ldots\}$. The approximate limiting values $\mathfrak{w}_{\rm lim}$ and $\mathfrak{q}_{\rm lim}$ for $n_s = 0, 1, 2, \ldots, 10$ are shown in table 1.

The existence of a meeting point between two contiguous dispersion relation curves suggests that for the special wavenumber values $\mathfrak{q} = \mathfrak{q}_{\rm lim}$ the corresponding quasinormal frequencies $\mathfrak{w} = -i\mathfrak{w}_{\rm lim}$ represent double poles of the CFT current-current correlation functions. A strong support to such a conclusion comes from the behavior of the connection coefficients $\mathcal{A}_{(j)}(\mathfrak{w}, \mathfrak{q})$ as a function of $\mathfrak{w} = \mathfrak{w}_R - i\mathfrak{w}_I$ for small values of $\mathfrak{q}$ and $\mathfrak{w}_R = 0$. In

[5] In this specific case, the time of computation spent by the numerical code based on the Horowitz-Hubeny method [31] written to find the quasinormal frequencies becomes very large as ϵ approaches zero.

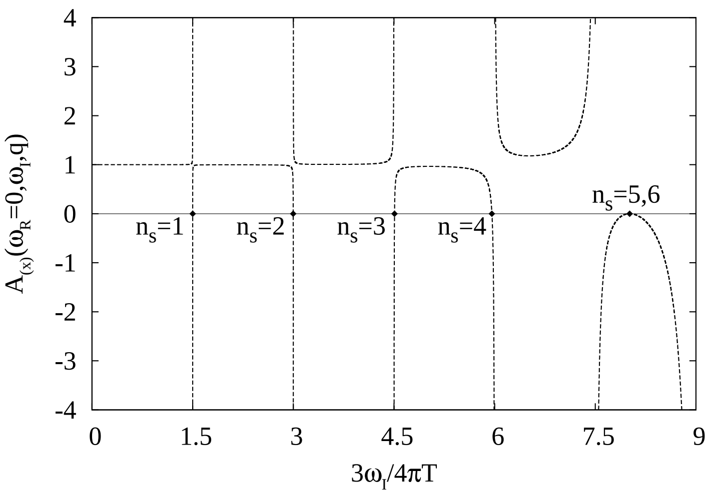

Figure 3: The connection coefficient $\mathcal{A}_{(x)}(\mathfrak{w},\mathfrak{q})$ for $\mathfrak{w}_R = 0$, $\mathfrak{w}_I = (0,9)$ and $\mathfrak{q} = \mathfrak{q}_{\rm lim} \simeq 0.0132$. The points represent the $n_s = 1, 2, .., 6$ purely imaginary quasinormal frequencies associated to the axial electromagnetic perturbations. Note the coincidence of the points corresponding to $n_s = 5$ and $n_s = 6$, indicating a possible doubleness of the related quasinormal frequency.

figure 3 it is shown the profile of $\mathcal{A}_{(x)}(\mathfrak{w},\mathfrak{q})$ for $\mathfrak{q} = \mathfrak{q}_{\rm lim} \simeq 0.0132$ and $0 < \mathfrak{w}_I < 9$. For this wavenumber, there are six purely imaginary quasinormal frequencies encompassing the $n_s = 1$ to $n_s = 6$ axial QNM, which by definition are the points where $\mathcal{A}_{(x)}(\mathfrak{w},\mathfrak{q}) = 0$. In particular, for $n_s = 5$ and $n_s = 6$, the zeros of $\mathcal{A}_{(x)}(\mathfrak{w},\mathfrak{q})$ coincide, indicating that the corresponding quasinormal frequencies are identical (see also figure 2).

The multiplicity of specific quasinormal frequencies as poles of the correlation functions may also be verified through the derivatives of $\mathcal{A}_{(j)}(\mathfrak{w},\mathfrak{q})$ with respect to $\mathfrak{w}$. This was done numerically, by taking fixed values of $\mathfrak{q}$ and letting $\mathfrak{q} \to \mathfrak{q}_{\rm lim}$. It was found that the first derivative of $\mathcal{A}_{(j)}(\mathfrak{w},\mathfrak{q})$ in relation to $\mathfrak{w}$ vanishes when $\mathfrak{w} = -i\mathfrak{w}_{\rm lim}$ and $\mathfrak{q} = \mathfrak{q}_{\rm lim}$, but it is not zero for other values of the wavenumber. This result proves that $\mathfrak{w} = -i\mathfrak{w}_{\rm lim}$ corresponds to, at least, a double zero of $\mathcal{A}_{(j)}(\mathfrak{w},\mathfrak{q})$, and consequently, to a double pole of the corresponding current-current correlation functions.

3.5.2 Ordinary quasinormal modes

The electromagnetic perturbations of AdS black holes present also a family of regular (ordinary) quasinormal modes whose frequencies have nonzero real and imaginary parts. The numerical results for the quasinormal frequencies of the first five regular modes are shown respectively in figures 4 and 5 for axial and polar fluctuations.

The form of the dispersion relations $\mathfrak{w}_R(\mathfrak{q})$ and $\mathfrak{w}_I(\mathfrak{q})$ indicates a connection between the electromagnetic ordinary QNM and the family of purely damped modes discussed in the last section. As shown in figures 4 and 5, each regular quasinormal frequency only appears for $\mathfrak{q}$ larger than a minimum wavenumber value, which (to a good approximation) coincides with the limiting value of the corresponding pair of purely damped modes, $\mathfrak{q}_{\rm lim}$. That is to say, all the dispersion relations, $\mathfrak{w}(\mathfrak{q})$, for the ordinary QNM begin at the points $(\mathfrak{w}_{\rm lim}, \mathfrak{q}_{\rm lim})$,

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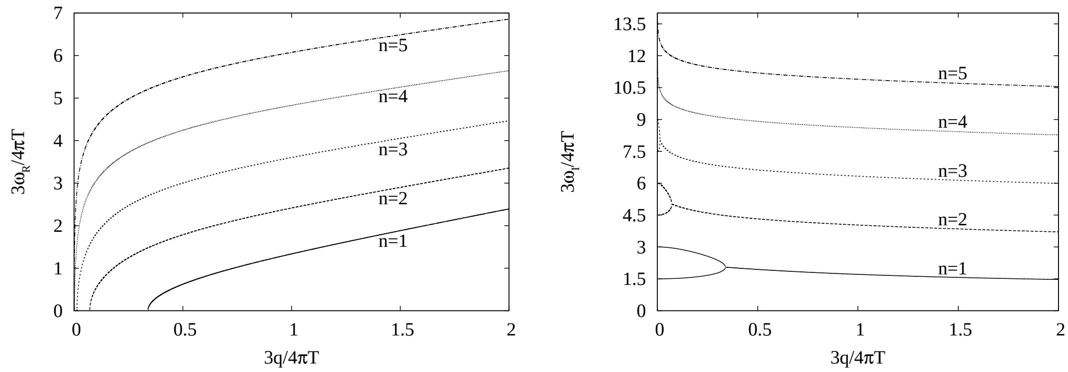

Figure 4: The real (left) and imaginary (right) parts of the frequencies $\mathfrak{w} = 3\omega/4\pi T$ for the first five ordinary axial electromagnetic modes as a function of the normalized wavenumber $\mathfrak{q} = 3q/4\pi T$. The quantum number n arranges the regular polar QNM in increasing order of values of $\mathfrak{w}_I$. In the right it is also shown the frequencies $\mathfrak{w}_s = 3\omega_s/4\pi T$ associated to the axial electromagnetic purely damped QNM.

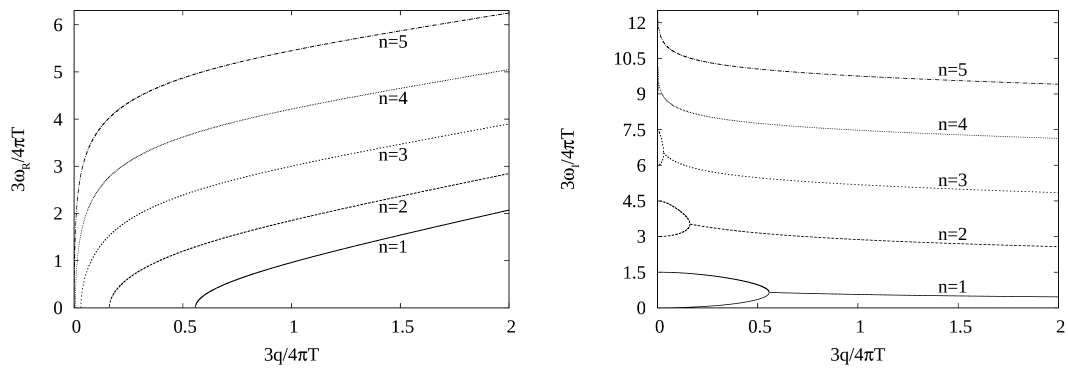

Figure 5: The real (left) and imaginary (right) parts of the frequencies $\mathfrak{w} = 3\omega/4\pi T$ for the first five ordinary polar electromagnetic modes as a function of the normalized wavenumber $\mathfrak{q} = 3q/4\pi T$. As for the axial modes, the quantum number n arranges the regular QNM from lower to higher values of $\mathfrak{w}_I$. In the right it is also shown the frequencies $\mathfrak{w}_s = 3\omega_s/4\pi T$ associated to the polar electromagnetic purely damped QNM.

with the real parts starting from zero value, $\mathfrak{w}_R(\mathfrak{q} \to \mathfrak{q}_{\rm lim}^+) = 0$, while the imaginary parts start at $\mathfrak{w}_I(\mathfrak{q} \to \mathfrak{q}_{\rm lim}^+) = \mathfrak{w}_{\rm lim}$. For higher wavenumber values, the ordinary electromagnetic modes show a sequence of quasinormal frequencies whose imaginary parts grow with the principal quantum number n. In this respect, figures 4 and 5 show a similar behavior for the real parts of the quasinormal electromagnetic frequencies.

The quasinormal frequencies found here show that AdS black holes are not good oscillators. As it is well known, an interesting way of measuring the quality of an oscillator is by means of its quality factor $Q = \mathfrak{w}_R/2\mathfrak{w}_I$. In general, in the region of small wavenumbers, the electromagnetic QNM have very small quality factors, $Q \ll 1$. For instance, taking $\mathfrak{q} = 0.557319$ and considering the fundamental polar mode one obtains $Q = 7.24 \times 10^{-5}$, a quality factor typical to highly damped oscillatory systems. On the other hand, quality factors of the order of unity are found for larger wavenumber values, such as $Q = 1$ for $\mathfrak{q}$

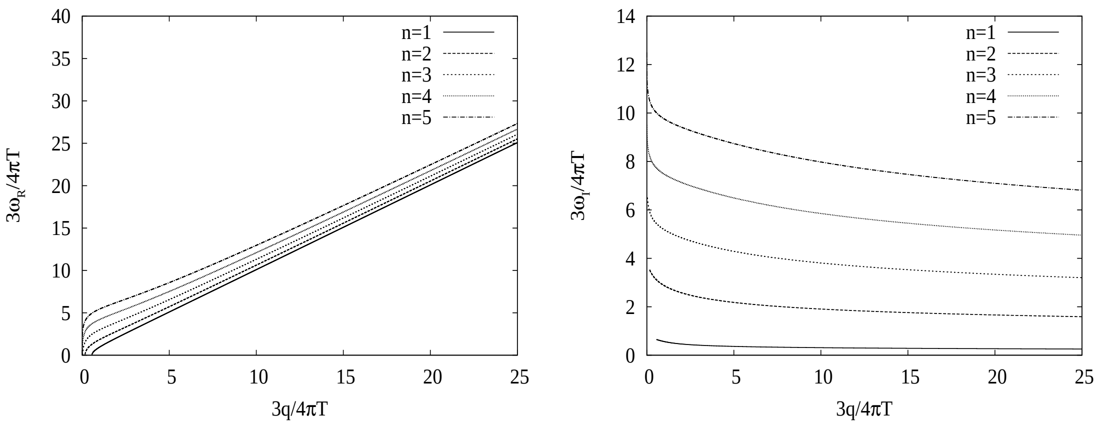

Figure 6: Graphs of the dispersion relations of the first five regular polar electromagnetic QNM for large values of $\mathfrak{q}$. The figure on the left hand side shows the real part of the frequency, $\mathfrak{w}_R$, while the graph on the right hand side is for the imaginary part, $\mathfrak{w}_I$.

around 1.11, and $Q = 85.9$ for $\mathfrak{q} = 40$.

From the holographic field theory point of view, the real part of the frequencies may be interpreted as quasiparticle excitation energies in the dual plasma defined at the conformal boundary of the AdS spacetime. However, such an interpretation only makes sense for excitations, or quasinormal modes, with large quality factors. In fact, according to Heisenberg uncertainty principle, the uncertainty in the energy of a quasiparticle is of the order of ω_I (in units of $\hbar$). Hence, quality factors smaller than unity imply in energy uncertainties larger than the energies of the quasiparticles themselves, making the interpretation of energy excitations as quasiparticles meaningless.

In the $Q \gg 1$ regime, where the quasiparticle interpretation is feasible, figure 6 shows that the dispersion relations of the ordinary electromagnetic QNM frequencies have real parts approaching straight lines of the form $\mathfrak{w}_R = \mathfrak{q} + \mathfrak{b}_n$, where $\mathfrak{b}_n$ depends only on the specific mode n. This means the functions $\omega_R(q)$ approach the usual energy-momentum relation associated to a zero rest mass particle, $\omega_R = q$, as $T \to 0$. Furthermore, the characteristic damping time ($\tau = 1/\omega_I$) of the electromagnetic fluctuations diverges in the limit $\mathfrak{q} \to \infty$, i.e., the functions $\omega_I(q)$ tend to zero for large wavenumbers. All of the above results are consistent with the expected properties of poles of correlation functions in quantum field theories at zero temperature [67].

4. Gravitational quasinormal modes

Even though gravitational perturbations of plane-symmetric black holes in $(3+1)$-dimensional AdS spacetimes have been analyzed in some extent [63, 64, 53 − 55], the quasinormal-mode dispersion relations for the KS variables were not found yet, and hence the comparison with the spectra obtained by using the RWZ gauge-invariant variables was not performed. This is done next.

4.1 Fundamental equations for gravitational fluctuations

As usual, gravitational perturbations are described here in terms of linear metric fluctuations, which means the metric for the perturbed spacetime is written as $g_{MN} = g^0_{MN} + h_{MN}$, where h_{MN} is considered as a perturbation in the background metric g^0_{MN}, given by eq. (2.6). Among the variety of possible gauge choices in studying metric perturbations, an interesting choice is the so-called radial gauge, in which the coordinate system is chosen in such a way that $h_{rt} = h_{rx} = h_{ry} = h_{rr} = 0$. Since one of the aims here is to investigate the relation among the perturbations of plane-symmetric AdS_4 black holes and the correlation functions in the dual CFT, it is convenient to use the radial gauge formalism to study the metric fluctuations. A brief description of this formalism is given in the following.

As it was done in the previous section when studying the electromagnetic perturbations, the isometries of spacetime (2.6) allow Fourier transforming coordinates t, x and y, and writing metric fluctuations h_{MN} as

$$h_{MN}(t,x,y,r) = \frac{1}{(2\pi)^3}\int d\omega\, dk_x\, dk_y\, e^{-i\omega t + ik_x x + ik_y y}\widetilde{h}_{MN}(\omega, k_x, k_y, r). \tag{4.1}$$

Again the wave three-vector may be chosen as $k_\mu = (-\omega, 0, q)$, and hence metric perturbations can be split into two disjoint sets. Namely, the axial (transverse) sector of gravitational perturbations is characterized by the quantities h_{tx} and h_{yx}, and the polar (longitudinal) sector of perturbations is composed by h_{tt}, h_{xx}, h_{yy}, and h_{ty}.

4.1.1 Axial perturbations in the radial gauge

Now one needs to find evolution equations for the axial (transverse) gravitational perturbations in the radial gauge, that are composed by the metric fluctuations h_{tx} and h_{yx}. The task is carried out following the usual procedure of the theory of linear perturbations. The linearized Einstein equations corresponding to the axial sector of gravitational perturbations yield a set of three coupled differential equations for h_{tx} and h_{yx} [63]. Of course, such equations are not independent from each other, and one of them may be eliminated as a combination of the other two. A resulting system of linearly independent equations (after Fourier transforming them) which is interesting for the present study is the following:

$$H'_{tx} + \frac{\mathfrak{q}\mathfrak{h}}{\mathfrak{w}}H'_{yx} = 0, \tag{4.2}$$

$$H''_{tx} - \frac{2}{u}H'_{tx} - \frac{\mathfrak{q}}{\mathfrak{h}}\left(\mathfrak{w}H_{yx} + \mathfrak{q}H_{tx}\right) = 0, \tag{4.3}$$

where H_{tx} and H_{yx} are defined by

$$H_{tx} = \frac{L^2}{r^2}\,h_{tx}, \qquad\qquad H_{yx} = \frac{L^2}{r^2}\,h_{yx}. \tag{4.4}$$

As in the case of electromagnetic perturbations, a mandatory condition that a candidate for fundamental variable must satisfy is being gauge invariant, which for metric perturbations means the candidate has to be invariant under infinitesimal coordinate transformations. Inspired once again in the work by Kovtun and Starinets [60], among

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the different combinations of axial functions H_{tx} and H_{yx} which furnish gauge-invariant quantities, one takes

$$Z_1 = i\,(qH_{tx} + \omega H_{yx}) \tag{4.5}$$

as the fundamental gauge-invariant function of the axial gravitational perturbations.

Decoupling the system of differential equations (4.2) and (4.3) in terms of the fundamental variable Z_1, it results the solely second-order differential equation

$$Z_1'' - \frac{2(\mathfrak{w}^2 - \mathfrak{q}^2\mathfrak{h})\mathfrak{h} - u\mathfrak{h}'\mathfrak{w}^2}{u\mathfrak{h}(\mathfrak{w}^2 - \mathfrak{q}^2\mathfrak{h})} Z_1' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2\mathfrak{h}}{\mathfrak{h}^2} Z_1 = 0. \tag{4.6}$$

Solutions to this equation satisfying the QNM boundary conditions are studied below.

4.1.2 Polar perturbations in the radial gauge

The polar (longitudinal) sector of gravitational perturbations in the radial gauge is described by metric fluctuations h_{tt}, h_{xx}, h_{yy} and h_{ty}. These components of the metric perturbation tensor are used to define new quantities

$$H_{tt} = \frac{1}{f}\,h_{tt}, \qquad H_{xx} = \frac{L^2}{r^2}\,h_{xx}, \qquad H_{yy} = \frac{L^2}{r^2}\,h_{yy}, \qquad H_{ty} = \frac{L^2}{r^2}\,h_{ty}, \tag{4.7}$$

which are more appropriate to deal with during calculations to obtain perturbation equations. Hence, the polar components of linearized Einstein equations furnish a set of seven coupled equations for the variables defined in eqs. (4.7). Only four of such a set are independent equations and, among the possible choices, the more interesting for the present work are

$$H_{ty}' = \frac{2u\mathfrak{w}\mathfrak{q}}{b(u)}\,(H_{xx} - H_{tt}) + \frac{ua(u)}{2\mathfrak{q}\mathfrak{h}b(u)}\,(\mathfrak{w}H_{xx} + \mathfrak{w}H_{yy} + 2\mathfrak{q}H_{ty}) - \frac{4\mathfrak{w}\mathfrak{h}}{\mathfrak{q}b(u)} H_{tt}', \tag{4.8}$$

$$H_{xx}' = \frac{\mathfrak{w}ua(u)}{2\mathfrak{q}^2\mathfrak{h}^2 b(u)}\,(\mathfrak{w}H_{xx} + \mathfrak{w}H_{yy} + 2\mathfrak{q}H_{ty}) - \frac{c(u)}{\mathfrak{q}^2 b(u)} H_{tt}' + \frac{2u\mathfrak{w}^2}{\mathfrak{h}b(u)} H_{xx} + \frac{ua(u)}{2\mathfrak{h}b(u)} H_{tt}, \tag{4.9}$$

$$\begin{aligned} H_{yy}' = & \frac{2u}{\mathfrak{h}b(u)}\left[\mathfrak{w}^2 H_{xx} + \mathfrak{w}^2 H_{yy} + \mathfrak{q}^2\mathfrak{h}\,(H_{tt} - H_{xx}) + 2\mathfrak{q}\mathfrak{w}H_{ty}\right] + \left(\frac{4\mathfrak{h}}{b(u)} + \frac{c(u)}{\mathfrak{q}^2 b(u)}\right) H_{tt}' \\ & - \frac{u}{2\mathfrak{h}b(u)}\left[4\mathfrak{w}^2 H_{xx} + c(u)H_{tt}\right] - \frac{\mathfrak{w}uc(u)}{2\mathfrak{q}^2\mathfrak{h}^2 b(u)}\,(\mathfrak{w}H_{xx} + \mathfrak{w}H_{yy} + 2\mathfrak{q}H_{ty}), \end{aligned} \tag{4.10}$$

$$H_{tt}'' = \frac{2\mathfrak{w}^2}{\mathfrak{h}b(u)}\,(H_{xx} + H_{yy}) + \frac{2\mathfrak{q}}{\mathfrak{h}b(u)}\,(2\mathfrak{w}H_{ty} + \mathfrak{q}\mathfrak{h}H_{tt}) + \frac{(2+u^3)}{2u\mathfrak{h}b(u)}\left[\mathfrak{q}^2 H_{xx} + (8+u^3)H_{tt}'\right], \tag{4.11}$$

where, as above, the primes denote derivatives with respect to the variable $u = r_0/r$, and coefficients $a(u) = 3u^4 - 12u - 4\mathfrak{w}^2$, $b(u) = \mathfrak{h} + 3$, and $c(u) = 4\mathfrak{w}^2 - \mathfrak{q}^2 b(u)$ were introduced to simplify notation.

Finally, a gauge-invariant function Z_2 is built as a particular combination of the metric perturbations

$$Z_2 = 4\omega qH_{ty} + 2\omega^2 H_{yy} + \left[q^2(3-\mathfrak{h}) - 2\omega^2\right] H_{xx} + 2q^2\mathfrak{h}H_{tt}, \tag{4.12}$$

for which, uncoupling the equations of motion (4.8)–(4.11), it is found the following second-order differential equation

$$Z_2'' - \frac{4\mathfrak{w}^2(2+u^3)+\mathfrak{q}^2 d(u)}{u\mathfrak{h}c(u)} Z_2' + \frac{4\mathfrak{w}^4+\mathfrak{q}^4\mathfrak{h}b(u)-\mathfrak{q}^2 e(u)}{\mathfrak{h}^2 c(u)} Z_2 = 0, \tag{4.13}$$

where $d(u) = 4u^3 - 5u^6 - 8$ and $e(u) = 9u^4\mathfrak{h} + \mathfrak{w}^2(8-5u^3)$. Equations (4.6) and (4.13) are the fundamental equations that are going to be used in the next sections to compute the QNM spectra associated to gravitational perturbations of plane AdS_4 black holes.

4.2 Stress-energy tensor correlation functions

For the gravitational perturbations, the AdS/CFT correspondence establishes a relation among the solutions of eqs. (4.6) and (4.13) and the stress-energy tensor of the dual CFT. From this relation, the stress-energy tensor correlators can be determined, and in order to do that the explicit form of the fields in the bulk AdS spacetime has to be known. More precisely, in order to impose the ingoing-wave condition at the horizon, and to map AdS to CFT quantities at the boundary of the spacetime, the asymptotic form of the metric perturbation functions close to the horizon and at the boundary are necessary.

In the horizon neighborhood ($u \approx 1$), the gravitational gauge-invariant variables Z_1 and Z_2 have a similar behavior as the electric field components (see section 3.2), viz, $Z_{1,2} \sim \mathfrak{h}^{\pm i\mathfrak{w}/3}$. As in the electromagnetic case, to compute the retarded Green functions, one has to choose the solutions corresponding to the negative imaginary power, $Z_{1,2} \sim \mathfrak{h}^{-i\mathfrak{w}/3}$. On the other hand, at the conformal boundary of the AdS spacetime, the metric fluctuations are such that

$$Z_1 = \mathcal{A}_{(1)}(\mathfrak{w},\mathfrak{q}) + \cdots + \mathcal{B}_{(1)}(\mathfrak{w},\mathfrak{q})u^3 + \cdots, \tag{4.14}$$

$$Z_2 = \mathcal{A}_{(2)}(\mathfrak{w},\mathfrak{q}) + \cdots + \mathcal{B}_{(2)}(\mathfrak{w},\mathfrak{q})u^3 + \cdots, \tag{4.15}$$

where ellipses denote higher powers of u, and quantities $\mathcal{A}_{(1)}(\mathfrak{w},\mathfrak{q})$ and $\mathcal{B}_{(1)}(\mathfrak{w},\mathfrak{q})$, and $\mathcal{A}_{(2)}(\mathfrak{w},\mathfrak{q})$ and $\mathcal{B}_{(2)}(\mathfrak{w},\mathfrak{q})$ are the connection coefficients related to the differential equations (4.6) and (4.13), respectively.

For the remaining of this section, as usual, the gravitational perturbations are split into axial and polar sectors and the analysis of the corresponding actions, coming from eq. (2.9), are performed separately for both of the perturbation types.

4.2.1 Axial sector

It is well known that in the calculation of two-point correlation functions from the gravitational action only quadratic terms in metric perturbations need to be considered. Moreover, according to the Lorentzian AdS/CFT prescription [37] (see also ref. [80]), in order to obtain the CFT retarded correlators the relevant terms are the quadratic terms in the derivatives of $H_{\mu\nu}$. Hence, collecting all of the contributions coming from the gravitational part of action (2.9), one gets

$$S^{(2)} = \frac{P}{2}\int du\, d^3x \frac{1}{u^2}\left[H_{tx}'^2 - \mathfrak{h}H_{yx}'^2\right] + \cdots, \tag{4.16}$$

where

$$P = \left(\frac{4\pi T}{3}\right)^3 \frac{L^2}{2\kappa_4^2} = \frac{8\sqrt{2}}{81}\pi^2 N^{3/2} T^3 \tag{4.17}$$

is interpreted as the pressure of the dual plasma [64].

Now expressing functions $H_{tx}^{'}$ and $H_{yx}^{'}$ in terms of the axial fundamental variable Z_1 through eqs. (4.2)–(4.5), substituting the resulting relations into eq. (4.16), and making use of the fundamental equation (4.6), it is found the following action (at the boundary)

$$S_{boundary}^{(2)} = \frac{P}{2} \lim_{u\to 0} \int \frac{d\mathfrak{w}\, d\mathfrak{q}}{(2\pi)^2} \frac{\mathfrak{h}}{u^2(\mathfrak{w}^2 - \mathfrak{q}^2\mathfrak{h})} Z_1^{'}(u,k) Z_1(u,-k) + S_{CT}^{(2)}, \tag{4.18}$$

where the contact terms represented by $S_{CT}^{(2)}$ do not carry derivatives of the metric perturbation functions. In the calculation of the correlation functions, after Fourier transformation, the contact terms give rise to derivatives of the Dirac delta function. Their removal can be done through the holographic renormalization, with the inclusion of appropriate counter terms in the supergravity action [81].

Besides using eq. (4.5), the asymptotic expansion given by eq. (4.14) is used to write the derivative of the gauge-invariant quantity Z_1 in terms of the boundary values of the perturbation fields $H_{\mu\nu}^0(k) = H_{\mu\nu}(u \to 0, k)$, and then the AdS/CFT prescription [37] can be applied to the present case in order to calculate the retarded correlation functions of the holographic stress-energy tensor $T^{\mu\nu}$. The result is

$$G_{tx,tx} = -3P \frac{\mathfrak{q}^2}{(\mathfrak{w}^2 - \mathfrak{q}^2)} \frac{\mathcal{B}_{(1)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(1)}(\mathfrak{w},\mathfrak{q})}, \tag{4.19}$$

$$G_{tx,yx} = 3P \frac{\mathfrak{w}\mathfrak{q}}{(\mathfrak{w}^2 - \mathfrak{q}^2)} \frac{\mathcal{B}_{(1)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(1)}(\mathfrak{w},\mathfrak{q})}, \tag{4.20}$$

$$G_{yx,yx} = -3P \frac{\mathfrak{w}^2}{(\mathfrak{w}^2 - \mathfrak{q}^2)} \frac{\mathcal{B}_{(1)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(1)}(\mathfrak{w},\mathfrak{q})}. \tag{4.21}$$

As it happens for the current-current correlations functions, one can find general expressions for the two-point thermal functions associated to the stress-energy tensor which hold for any scale invariant $(2+1)$-dimensional field theory (see appendix A). For fluctuations of the transverse momentum density in the CFT one has the correlators

$$G_{tx,tx} = \frac{\mathfrak{q}^2}{2\,(\mathfrak{w}^2 - \mathfrak{q}^2)} G_1(\mathfrak{w},\mathfrak{q}), \tag{4.22}$$

$$G_{tx,yx} = -\frac{\mathfrak{w}\mathfrak{q}}{2\,(\mathfrak{w}^2 - \mathfrak{q}^2)} G_1(\mathfrak{w},\mathfrak{q}), \tag{4.23}$$

$$G_{yx,yx} = \frac{\mathfrak{w}^2}{2\,(\mathfrak{w}^2 - \mathfrak{q}^2)} G_1(\mathfrak{w},\mathfrak{q}). \tag{4.24}$$

Therefore, by comparing the general expressions of eqs. (4.22)–(4.24) to the results given in eqs. (4.19)–(4.21), the following scalar function is found

$$G_1(\mathfrak{w},\mathfrak{q}) = -6P \frac{\mathcal{B}_{(1)}(\mathfrak{w},\mathfrak{q})}{\mathcal{A}_{(1)}(\mathfrak{w},\mathfrak{q})}. \tag{4.25}$$

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It is then seen that Dirichlet condition imposed on the fundamental variable Z_1 at the boundary, $Z_1(0) = \mathcal{A}_{(1)}(\mathfrak{w}, \mathfrak{q}) = 0$, leads straightforwardly to the poles of the correlation functions $G_{tx,tx}$, $G_{tx,yx}$ and $G_{yx,yx}$. As a consequence of this result, such a requirement also yields the quasinormal spectrum associated to the axial gravitational perturbation modes of plane-symmetric black holes.

4.2.2 Polar sector

The procedure to be applied to the polar sector of gravitational perturbations is the same as for the axial sector. The starting point here is the part of the boundary gravitational action built with the quadratic terms in the polar metric perturbations, which is given by [64]

$$S^{(2)}_{boundary} = \frac{P}{2} \lim_{u\to 0} \int d^3x \bigg[\frac{1}{4} \left(2H_{tt}^2 - 8H_{ty}^2 + H_{tt}H_{xx} + H_{tt}H_{yy} \right) - \frac{1}{4} \left(H_{xx} - H_{yy} \right)^2 - \frac{\mathfrak{h}}{2u^2} \left(H_{ty}^2 + H_{xx}H_{yy} - H_{tt}H_{xx} - H_{tt}H_{yy} \right)' \bigg]. \tag{4.26}$$

By using the relation among polar metric fluctuations and the gauge-invariant variable Z_2, eq. (4.12), and the equations of motion (4.8)–(4.11), the boundary action (4.26) can be cast into the form

$$S^{(2)}_{boundary} = \frac{P}{2} \lim_{u\to 0} \int \frac{d\mathfrak{w} d\mathfrak{q}}{(2\pi)^2} \frac{\mathfrak{h}}{u^2 \left[4\mathfrak{w}^2 - \mathfrak{q}^2(4 - u^3) \right]^2} Z_2'(u,k) Z_2(u,-k) + S^{(2)}_{CT}, \tag{4.27}$$

where the contact terms $S^{(2)}_{CT}$ do not contain derivatives of metric perturbations. One now uses the asymptotic expansion (4.15) to write the derivative of the gauge-invariant variable Z_2 in terms of boundary values of the polar metric perturbations $H^0_{\mu\nu}(k)$. After substituting the resulting expression into the action (4.27), the appropriate functional derivatives[6] of the action with respect to the independent fields $H^0_{tt}(k)$, $H^0_{ty}(k)$, $H^0_{yy}(k)$ and $H^0_{xx}(k)$ are performed. Notice, however, that in the case of polar gravitational perturbations, the use of the AdS/CFT prescription is not direct. In fact, it is first necessary to identify explicitly how the metric perturbations couple to the stress-energy tensor at the boundary. As discussed in refs. [82, 83], such a coupling is given by

$$-\frac{1}{2} \int dt\, d^2x\, h^\nu{}_\mu T^\mu{}_\nu = -\frac{1}{2} \int dt\, d^2x \left[H^0_{tt} T^{tt} + H^0_{xx} T^{xx} + H^0_{yy} T^{yy} + 2H^0_{ty} T^{ty} \right]. \tag{4.28}$$

Taking this coupling into account, the covariant components of the polar correlation functions are found

$$G_{\mu\nu,\alpha\beta} = Q_{\mu\nu,\alpha\beta} G_2(\mathfrak{w}, \mathfrak{q}), \tag{4.29}$$

where the scalar function $G_2(\mathfrak{w}, \mathfrak{q})$ is given by

$$G_2(\mathfrak{w}, \mathfrak{q}) = -6P \frac{\mathcal{B}_{(2)}(\mathfrak{w}, \mathfrak{q})}{\mathcal{A}_{(2)}(\mathfrak{w}, \mathfrak{q})} + \text{contact terms}, \tag{4.30}$$

and tensor $Q_{\mu\nu,\alpha\beta}$ is given in appendix A (see also [60]). This result shows definitely that Dirichlet boundary condition imposed on the fundamental variable Z_2 at infinity leads to the poles of the function $G_2(\mathfrak{w}, \mathfrak{q})$, and, by definition, to the quasinormal frequencies of polar gravitational vibration modes.

[6] Functional derivatives in the sense defined by the Lorentzian AdS/CFT prescription of ref. [37].

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4.3 QNM and the gauge-invariant variables

As in the study of electromagnetic quasinormal modes, a comparison between results found using RWZ variables with the ones obtained using KS gauge-invariant variables to describe gravitational perturbations deserves to be made. As in that case, while the axial quasinormal spectrum is independent of the choice of the fundamental variable, polar quasinormal spectrum strongly depends on it.

A strong evidence that the QNM spectra obtained using either RWZ or KS variables to describe axial gravitational perturbations are identical comes from the study of the hydrodynamic limit of such perturbations as performed in ref. [54] and in section 4.4 (see below). As a matter of fact, even though different methods have been employed in each case, both of the quasinormal spectra present a typical hydrodynamic shear mode with diffusion coefficient $D = 1/4\pi T$, independently if one uses variable $Z^{(-)}$, or variable Z_1. Furthermore, it can be shown that the explicit relation between RWZ and KS variables is

$$Z_1 = \frac{f}{r^2}\partial_r\left[rZ^{(-)}\right], \tag{4.31}$$

and at the AdS spacetime boundary, axial fundamental variables $Z^{(-)}$ and Z_1 are proportional to each other,

$$Z_1(u)\big|_{u=0} = \frac{1}{L^2}Z^{(-)}(u)\big|_{u=0}, \tag{4.32}$$

what proves that the two spectra obtained from $Z^{(-)}$ and Z_1 are indeed identical to each other.

The situation in the polar sector is quite diverse from what happens in the axial sector. As it is shown in the sequence of the present work (see sections 4.4 and 4.5), RWZ and KS variables with the same boundary conditions generate different quasinormal frequencies. In particular, it is shown in section 4.4 that the hydrodynamic limit of Z_2 contains a sound wave mode which is not seen in the quasinormal spectrum obtained from $Z^{(+)}$.

4.4 Dispersion relations for the hydrodynamic QNM

For the fluctuations of the stress-energy tensor in the dual field theory, hydrodynamics predicts a shear mode in the transverse (axial) sector and a sound wave mode in the longitudinal (polar) sector. As it is going to be shown below, these modes also appear in the gravitational QNM spectra of the plane black holes as long as one investigates the regime of small frequencies and wavenumbers, $\mathfrak{w} \to 0$ and $\mathfrak{q} \to 0$. Proceeding analogously to the case of electromagnetic perturbations studied in section 3.4, the following change of variables is done

$$H_j(u) = \mathfrak{h}^{i\mathfrak{w}/3}Z_j(u), \qquad j = 1, 2. \tag{4.33}$$

Functions H_1 and H_2 are then expanded in power series of $\mathfrak{w}$ and $\mathfrak{q}$. Besides being approximate solutions to eqs. (4.6) and (4.13) in the hydrodynamic limit, such series must represent ingoing waves at the horizon, namely, $H_j(u)\big|_{u=1} =$ constant. These conditions are fulfilled by the following expansions:

$$Z_1 = C_1\mathfrak{h}^{-i\mathfrak{w}/3}\left[1 + i\frac{\mathfrak{q}^2\mathfrak{h}}{3\mathfrak{w}} + \mathcal{O}(\mathfrak{w}^2)\right], \tag{4.34}$$

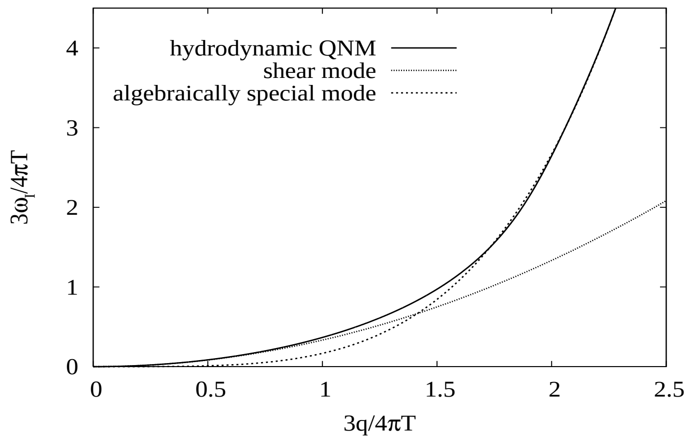

Figure 7: The dispersion relation (imaginary part) for the axial gravitational hydrodynamic mode (solid line), in comparison to the shear mode, $\mathfrak{q}^2/3$ (dotted line), and to the algebraically special mode, $\mathfrak{q}^4/6$ (dashed line).

$$Z_2 = C_2 \mathfrak{h}^{-i\mathfrak{w}/3} \left[2 - u^6 - 4\frac{\mathfrak{w}^2}{\mathfrak{q}^2} - \frac{4i\mathfrak{w}\mathfrak{h}}{3} + \mathcal{O}(\mathfrak{w}^2) \right], \tag{4.35}$$

where C_1 and C_2 are arbitrary normalization constants. Dirichlet conditions at the spacetime boundary, $u = 0$, are now imposed on both the axial and the polar fundamental variables, $Z_1(0) = 0$ and $Z_2(0) = 0$. The first variable leads to an axial quasinormal mode identified to the shear mode with

$$\omega = -iDq^2, \tag{4.36}$$

while the second variable furnishes a dispersion relation characteristic to a sound wave mode

$$\omega = \pm\frac{1}{\sqrt{2}}q - \frac{iDq^2}{2}, \tag{4.37}$$

where D is the diffusion coefficient, given by

$$D = \frac{\eta}{\varepsilon + P} = \frac{1}{4\pi T}, \tag{4.38}$$

with η and ε being respectively the shear coefficient and the energy density of the dual system.

Among other interesting results that can be found, combining eq. (4.38) to the thermodynamic Euler relation $P = -\varepsilon + Ts \Rightarrow s = (\varepsilon + P)/T$, one gets the ratio between the shear coefficient and the entropy density of the dual plasma,

$$\frac{\eta}{s} = \frac{\eta T}{\varepsilon + P} = \frac{1}{4\pi}, \tag{4.39}$$

or, in conventional units, $\eta/s = \hbar/4\pi k_B$. This ratio is the same for all finite temperature field theories with a dual gravitational description in the AdS spacetime [84]. It is also speculated that $\eta/s = 1/4\pi$ represents a lower bound –the KSS bound– of such a ratio for all fluids in nature.

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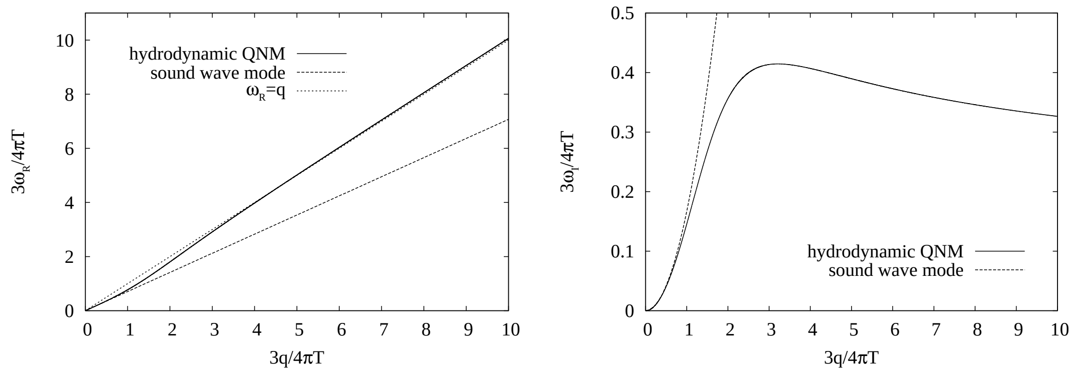

Figure 8: Real and imaginary parts, respectively, of the dispersion relation $\mathfrak{w}(\mathfrak{q})$ for the hydrodynamic longitudinal gravitational mode (solid lines). The dashed lines in both of the figures correspond to the sound wave mode, given by eq. (4.37), and the dotted curve in the figure on the left hand side is the relation $\mathfrak{w}_R = \mathfrak{q}$.

The complete dispersion relations for the gravitational hydrodynamic QNM are obtained by means of the Horowitz-Hubeny method [31], and, for the sake of simplicity, the numerical results for the axial and polar sectors are analyzed separately.

(i) *Axial modes*: The extension of the dispersion relation of the axial hydrodynamic QNM for large values of $\mathfrak{q}$ was already done in ref. [54], but for completeness it is also shown in figure 7. As the normalized wavenumber $\mathfrak{q}$ reaches values beyond of the hydrodynamic regime, the magnitude of the QNM frequency $\mathfrak{w} = -i\mathfrak{w}_I$ increases faster than the magnitude of the shear mode frequency $\mathfrak{w} = -i\mathfrak{q}^2/3$, and for very large wavenumber values, $\mathfrak{q} \gg 1$, the hydrodynamic QNM frequency approaches the algebraically special frequency $\mathfrak{w} = -i\mathfrak{q}^4/6$.

(ii) *Polar modes*: In this sector of perturbations, RWZ and KS variables submitted to the same boundary conditions generate different quasinormal frequencies. As seen above, the hydrodynamic limit of Z_2 presents a sound wave mode which is not present in the quasinormal spectrum obtained from $Z^{(+)}$. The extended dispersion relations for the longitudinal hydrodynamic QNM are shown in figure 8. The real part of the frequency clearly shows the transition from the hydrodynamic regime $\mathfrak{w}_R = \mathfrak{q}/\sqrt{2}$, at low wavenumbers, to a regime characterized by collisionless dual plasma in which the dispersion relation $\mathfrak{w}_R(\mathfrak{q})$ approaches the ultra-relativistic relation $\mathfrak{w}_R = \mathfrak{q}$. Between these two extreme regimes, the group velocity, defined by $c_s = d\mathfrak{w}_R/d\mathfrak{q}$, assumes values that are higher than the speed of light. The graphs in figure 9 show that $c_s > 1$ for all wavenumbers larger than $\mathfrak{q} \simeq 1.336$, and that the minimum decaying time (the maximum of $\mathfrak{w}_I$) corresponds to $\mathfrak{q} \simeq 3.213$. Notice, however, that $d\mathfrak{w}_R/d\mathfrak{q}$ surpasses the speed of light at wavenumber values lying outside the hydrodynamic regime and therefore that superluminal group velocity cannot be interpreted as the sound velocity in the corresponding media.

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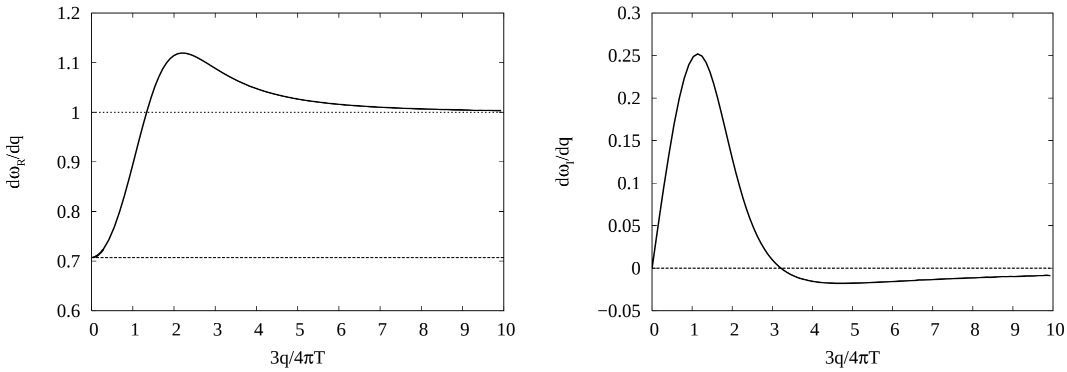

Figure 9: The group velocity $c_s = d\mathfrak{w}_R/d\mathfrak{q}$ and the derivative $d\mathfrak{w}_I/d\mathfrak{q}$ as a function of the normalized wavenumber $\mathfrak{q}$ for the polar hydrodynamic gravitational mode (solid lines). The dashed line in the figure on the left is the sound velocity for a (2+1)-dimensional CFT, $c_s = 1/\sqrt{2}$, and the dotted line in the same figure represents the speed of light, $c = 1$.

4.5 Dispersion relations for the non-hydrodynamic QNM

Conventionally, a non-hydrodynamic QNM is every mode for which the dispersion relation presents a gap in the limit $\mathfrak{q} \to 0$. That is to say, the quasinormal frequency $\mathfrak{w}(\mathfrak{q})$ tends to a nonzero value in the limit where the wavenumber goes to zero. These kind of gravitational perturbation modes are studied in this section.

As done in the case of electromagnetic perturbations, the QNM obtained by using RWZ gravitational variables $Z^{(\pm)}$, obeying Schrödinger-like equations, are compared to the QNM obtained by using the KS gauge-invariant quantities $Z_{1,2}$, which lead to the poles of stress-energy tensor correlators in the $\mathcal{N} = 8$ super-Yang-Mills field theory. In particular, the results obtained here from $Z_{1,2}$ are compared to the results of refs. [53, 54].

4.5.1 Purely damped modes

The spectra of gravitational perturbations of plane-symmetric AdS_4 black holes do not present non-hydrodynamic quasinormal frequencies with vanishing real part. In fact, the only purely damped mode of the gravitational perturbations is the axial hydrodynamic QNM which was already investigated in the last section.

4.5.2 Ordinary quasinormal modes

As usual, the study of the dispersion relations of regular non-hydrodynamic gravitational QNM is more conveniently performed by considering axial and polar sectors of such perturbations separately.

(i) *Axial modes*: As shown above, in the case of gravitational axial modes both of the fundamental variables Z_1 and $Z^{(-)}$ yield the same QNM spectrum. Even though a detailed study of the fundamental non-hydrodynamic QNM (based on the RWZ master variable) was performed in ref. [54], higher overtones of axial gravitational QNM were not fully investigated. Hence, the aim here is to complete the analysis by including such higher overtones. Figure 10 shows the numerical results for the dispersion relations of the first five axial quasinormal modes: $n = 1, \ldots, 5$. The general

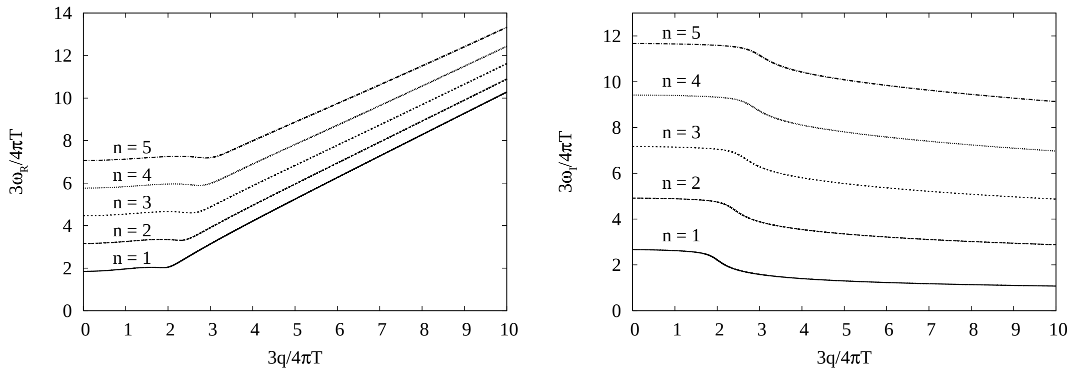

Figure 10: The first five quasinormal frequencies of non-hydrodynamic axial gravitational modes, $\mathfrak{w} = (3\omega_R/4\pi T) - i(3\omega_I/4\pi T)$, as a function of the normalized wavenumber $\mathfrak{q} = 3q/4\pi T$. The quantum number n arranges the modes in growing order according to the strength of the imaginary parts of the frequencies.

forms of the curves are approximately the same for all values of n: At intermediate values of $\mathfrak{q}$, there is a local minimum in the real part of the frequency $\mathfrak{w}_R(\mathfrak{q})$, and for large values of the wavenumber, every dispersion relation $\mathfrak{w}_R(\mathfrak{q})$ tends to some straight line parallel to the ultra-relativistic energy-momentum relation, $\mathfrak{w}_R = \mathfrak{q}$.

(ii) *Polar modes*: Contrary to the axial perturbations, the longitudinal gravitational fluctuations present different spectra as one takes Z_2 or $Z^{(+)}$ as fundamental variable. The numerical results for these modes, based on the gauge-invariant variable Z_2, are shown in figure 11. Notice that now the real part of the quasinormal frequency $\mathfrak{w}_R$ is a monotonic increasing function of $\mathfrak{q}$, showing neither local maxima nor local minima which, on contrary, appear in the quasinormal spectrum for the RWZ master variable $Z^{(+)}$ used in refs. [53, 54]. A comparison between the QNM spectrum found from the RWZ variable (cf. ref. [54]) and that found from the KS variable used here can also be done through the data presented in table 2. It is clearly seen the similarity between the two spectra in the two asymptotic regions of wavenumber values. Both of the spectra are approximately the same for small values and also for large values of $\mathfrak{q}$. The main differences happen in the regime where the normalized wavenumber $\mathfrak{q}$ is of the order of unity, which means that q is of the order of the blackhole Hawking temperature. Moreover, the real parts of the quasinormal frequencies in both of the spectra are relatively closer to each other when compared to the corresponding imaginary parts. This implies that, even though the QNM oscillation frequencies are essentially the same for both choices of variables, the decaying timescales ($\tau = 1/\omega_I$) are significantly smaller for the KS choice.

5. Final comments and conclusion

One of the important issues dealt with in the present work is related to the choice of appropriate variables in order to determine the QNM spectra of AdS black holes. It

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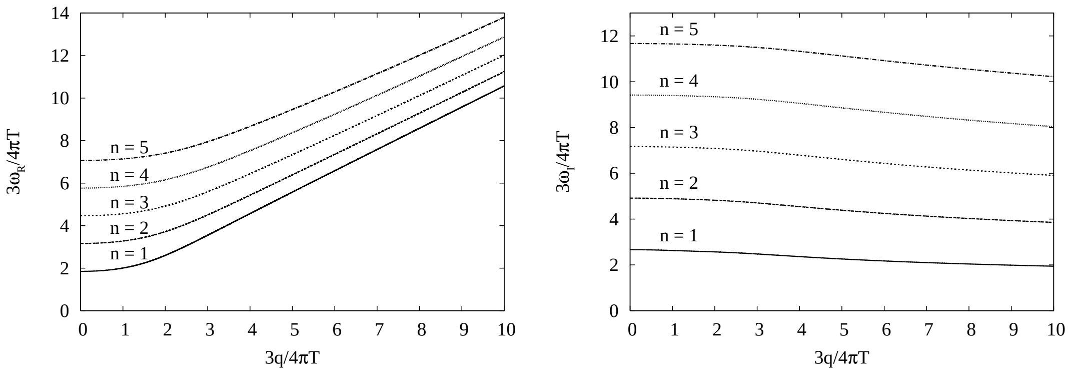

Figure 11: The first five quasinormal frequencies of non-hydrodynamic polar gravitational modes, $\mathfrak{w} = (3\omega_R/4\pi T) - i(3\omega_I/4\pi T)$, as a function of the normalized wavenumber $\mathfrak{q} = 3q/4\pi T$, ordered as in the case of axial gravitational modes.

	Kovtun-Starinets		Regge-Wheeler-Zerilli	
$\mathfrak{q}$	$\mathfrak{w}_R$	$\mathfrak{w}_I$	$\mathfrak{w}_R$	$\mathfrak{w}_I$
0.004	1.84942	2.66385	1.84945	2.66384
0.04	1.84964	2.66379	1.85027	2.66248
0.4	1.87207	2.65770	1.92488	2.52658
1	2.00603	2.62917	2.03016	1.92213
2	2.60256	2.56803	2.30526	1.55218
5	5.57791	2.25854	5.25618	1.27974
10	10.5703	1.94304	10.2839	1.07342

Table 2: Numerical results for the first non-hydrodynamic quasinormal mode associated to the polar gravitational perturbations. The second and third columns show respectively the values of real and imaginary parts of the frequencies obtained by using the KS variable Z_2, and the last two columns present the results obtained by using the RWZ variable $Z^{(+)}$, which are also shown in ref. [54].

is argued that Kovtun-Starinets gauge-invariant quantities, together with incoming-wave condition at horizon and Dirichlet boundary condition at infinity, should be used in order to find the correct quasinormal dispersion relations. The resulting spectrum for a given perturbation is in general different from what is obtained using other kind of variables with the same boundary conditions.

In the case of electromagnetic perturbations, contrary to what is obtained using the so-called Regge-Wheeler-Zerilli quantities $\Psi^{(\pm)}$ where the spectra for both the axial and the polar modes are the same, Kovtun-Starinets variables present a different spectrum for each mode type. Also, both of these sectors of electromagnetic perturbations present purely damped modes whose dispersion relations, in the limit of small wavenumbers, approach the bosonic Matsubara frequencies $\omega = -2i\pi T n_s$. This result can be compared to the "quasi-Matsubara" frequencies, $\omega = 2\pi T n_s(1-i)$, that have been found for zero wavenumber fluctuations of $(4+1)$-dimensional black branes [58]. The studies show the emergence of infinite sequences of bosonic Matsubara frequencies for both $(3+1)$- and

$(4+1)$-dimensional black branes, but the particular behavior of the QNM dispersion relations at zero wavenumber is quite different in each case. In $(3+1)$ dimensions the real part of the frequencies is zero for very small wavenumber values, while in $(4+1)$ dimensions real and imaginary parts of the frequencies are both finite at zero wavenumber. Moreover, as pointed out in section 3.5 and discussed in detail in ref. [67], the invariance of Maxwell equations under the electric field $\leftrightarrow$ magnetic field duality operation in $(3+1)$-dimensional spacetimes implies that there are no electromagnetic QNM at zero wavenumber. Such a duality invariance does not hold in higher dimensional spacetimes, what justifies the different behavior of zero wavenumber electromagnetic QNM found here when compared to the results of ref. [58].

Other special property of electromagnetic fluctuations deserving to be mentioned here is the cutoff in the dispersion relations of purely damped QNM at a particular value of the wavenumber, $\mathfrak{q} = \mathfrak{q}_{\text{lim}}$. This cutoff implies in an abrupt change in the behavior of the fundamental quasinormal mode, but not in the thermalization time τ. For wavenumbers in the interval $0 < \mathfrak{q} < \mathfrak{q}_{\text{lim}}$, parameter τ is given by the first purely damped mode, $\tau = 1/\omega_s$, while for wavenumber values above $\mathfrak{q}_{\text{lim}}$, the characteristic decaying time are governed by the fundamental ordinary QNM, $\tau = 1/\omega_I$. Since the imaginary parts of the frequencies of these modes are equal for $\mathfrak{q} = \mathfrak{q}_{\text{lim}}$, the thermalization time changes continuously for wavenumbers close to $\mathfrak{q}_{\text{lim}}$.

The numerical results for the gravitational QNM show that the thermalization time of axial modes for wavenumbers in the interval $0 < \mathfrak{q} < 1.935$ is determined by the hydrodynamic mode. This means that, at least in the limit $\mathfrak{q} \ll 1$, where the shear mode is a good approximation for the axial hydrodynamic QNM, the thermalization time τ is a linear function of Hawking temperature, $\tau \simeq 4\pi T/q^2$. As in the case of electromagnetic fluctuations, the transition to the regime where the thermalization time is determined by the first regular axial QNM is continuous. Such a transition happens for the values $\mathfrak{q} \simeq 1.935$ and $\mathfrak{w}_I \simeq 2.296$. Also, at this point the thermalization time reaches its minimum value, $\tau \simeq 0.104/T$. On the other hand, for polar gravitational perturbations the decaying time τ is always determined by the hydrodynamic QNM which reduces to the sound wave mode in the small wavenumber limit $\mathfrak{q} \ll 1$.

Finally, the behavior of the group velocity c_s shown in figure 9, which is greater than unity for all $\mathfrak{q} > 1.336$, deserves further analysis. First note that apparent superluminal propagation of this type, which at first sight seems to violate causality, has been found in other relativistic quantum systems. For instance, Scharnhorst [85] has shown in the case of Casimir effect that when vacuum fluctuations obey periodic boundary conditions, the two-loop corrections to the polarization tensor lead to superluminal photon propagation. Also, it is argued that the physically meaningful propagation velocity and, consequently, the one that defines the light cones in spacetime, is the front wave speed v_{wf}, which is given by the limit of the phase speed $v_{\text{ph}} = \omega/q$ when $\omega \to \infty$ (see, e.g., ref. [86] for a review). The graph on the left in figure 9 shows that v_{ph} approaches $c = 1$ at high frequencies, and, therefore, the oscillations in the super-Yang-Mills plasma do not violate causality.

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A. Correlators in a $(2+1)$-dimensional CFT

The Lorentz index structure of retarded Green functions of conserved currents and stress-energy tensor in a D-dimensional relativistic quantum field theory was discussed in details in ref. [60]. It was shown that the field theory correlators can be expressed in terms of a set of scalar functions. For the present work, a particularly interesting example is that of a $(2+1)$-dimensional finite temperature conformal field theory. In this specific case, with the wave three-vector in the form $k_\mu = (-\omega, 0, q)$, the transverse component of the current-current correlation functions can be written as

$$C_{x^1x^1}(k) = \Pi^T(\omega, q). \tag{A.1}$$

The longitudinal components in turn are given by

$$C_{tt}(k) = \frac{q^2}{(\omega^2 - q^2)}\Pi^L(\omega, q), \tag{A.2}$$

$$C_{tx^2}(k) = -\frac{\omega q}{(\omega^2 - q^2)}\Pi^L(\omega, q), \tag{A.3}$$

$$C_{x^2x^2}(k) = \frac{\omega^2}{(\omega^2 - q^2)}\Pi^L(\omega, q), \tag{A.4}$$

where $\Pi^T(\omega, q)$ and $\Pi^L(\omega, q)$ are two independent scalar functions. All the correlators of transverse momentum density are expressed in terms of a scalar function $G_1(\omega, q)$:

$$G_{tx^1,tx^1}(k) = \frac{1}{2}\frac{q^2}{(\omega^2 - q^2)}G_1(\omega, q), \tag{A.5}$$

$$G_{tx^1,x^1x^2}(k) = -\frac{1}{2}\frac{\omega q}{(\omega^2 - q^2)}G_1(\omega, q), \tag{A.6}$$

$$G_{x^1x^2,x^1x^2}(k) = \frac{1}{2}\frac{\omega^2}{(\omega^2 - q^2)}G_1(\omega, q). \tag{A.7}$$

In a similar way, the correlators of longitudinal momentum density, energy density, and diagonal stress are determined by another scalar function $G_2(\omega, q)$,

$$G_{\mu\nu,\alpha\beta}(k) = Q_{\mu\nu,\alpha\beta}(\omega, q)G_2(\omega, q), \tag{A.8}$$

where the components of the projector $Q_{\mu\nu,\alpha\beta}$ are given by:

$$Q_{tt,tt} = \frac{1}{2}\frac{q^4}{(\omega^2 - q^2)^2}, \qquad Q_{tt,tx^2} = -\frac{1}{2}\frac{\omega q^3}{(\omega^2 - q^2)^2}, \tag{A.9}$$

$$Q_{tt,x^1x^1} = -\frac{1}{2}\frac{q^2}{(\omega^2 - q^2)}, \quad Q_{tt,x^2x^2} = \frac{1}{2}\frac{\omega^2 q^2}{(\omega^2 - q^2)^2}, \tag{A.10}$$

$$Q_{x^1x^1,x^1x^1} = \frac{1}{2}, \quad Q_{x^1x^1,tx^2} = \frac{1}{2}\frac{\omega q}{(\omega^2 - q^2)}, \tag{A.11}$$

$$Q_{x^1x^1,x^2x^2} = -\frac{1}{2}\frac{\omega^2}{(\omega^2 - q^2)}, \quad Q_{x^2x^2,x^2x^2} = \frac{1}{2}\frac{\omega^4}{(\omega^2 - q^2)^2}, \tag{A.12}$$

$$Q_{x^2x^2,tx^2} = -\frac{1}{2}\frac{\omega^3 q}{(\omega^2 - q^2)^2}, \quad Q_{tx^2,tx^2} = \frac{1}{2}\frac{\omega^2 q^2}{(\omega^2 - q^2)^2}. \tag{A.13}$$

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